PC11 Lesson 7.4

Tuesday, May 9, 2017 12:12 PM

7.4 Reciprocal Functions

Warm up: For which values of x is each rational expression undefined? Find NPV's.

a) $\frac{1}{5x}$

$5x \neq 0$

$x \neq 0$

b) $\frac{1}{3x-5}$

$3x - 5 \neq 0$

$3x \neq 5$

$x \neq \frac{5}{3}$

c) $\frac{1}{2x^2-3x-2}$

$2x^2 - 3x - 2 \neq 0$

$(2x+1)(x-2) \neq 0$

$2x + 1 \neq 0$ $\quad x - 2 \neq 0$

$2x \neq -1$ $\quad x \neq 2$

$x \neq -\frac{1}{2}$

d) $\frac{1}{9x^2-16}$

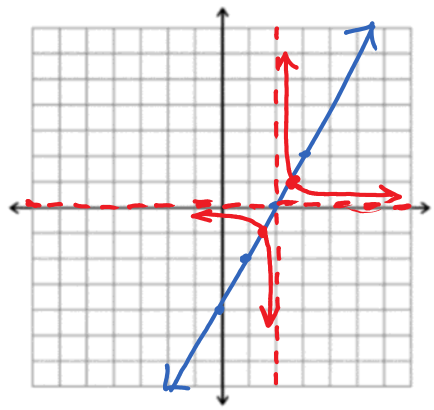
$9x^2 - 16 \neq 0$

$(3x+4)(3x-4) \neq 0$

$x \neq \pm\frac{4}{3}$

Investigation:

1. Graph the function $f(x) = 2x - 4$ and identify: slope
   a. X-intercept: $(2, 0)$
   b. Y-intercept: $(0, -4)$



2. Identify the non-permissible value(s) for the expression $\frac{1}{2x-4}$.

$2x - 4 \neq 0$

$2x \neq 4$

$x \neq 2$

3. To graph a reciprocal function such as $g(x) = \frac{1}{2x-4}$, there are several steps that need to be completed.
   a. Where would the non-permissible values occur on your graph? $x = 2$

   How does this relate to the graph of f(x)?

The graph of $\frac{1}{f(x)}$ cannot touch where $x = 2$.

x-intercepts of $f(x)$ = NPV's of $\frac{1}{f(x)}$ or $g(x)$

How would you indicate the non-permissible value(s) on your graph?

Dotted line a.k.a. asymptotes.

b. The vertical asymptote divides a graph into "regions". Because the asymptote represents a non-permissible value, the graph can approach the vertical asymptote, but NEVER cross it.

$\frac{1}{2x-4} = 0$

$1 \neq 0(2x-4)$

Can $g(x) = 0$? Why or why not?

No. $2x-4$ can never be zero and it's impossible to make $g(x)=0$ without that.

A **horizontal asymptote** is found at ___$y=0$ (x-axis)___.

$y = \frac{1}{y}$

$y^2 = 1$

$y = \pm\sqrt{1}$

$y = \pm 1$

c. Write the function $g(x)$ in terms of $f(x)$.

$g(x) = \frac{1}{f(x)}$

For which value(s) of y are f(x) and g(x) the same? ___$y = \pm 1$___

Would this be true for all functions and their reciprocals? Explain.

yes. The reciprocal of 1 is always 1, and the reciprocal of -1 is always -1.

These are considered the invariant points. ★

f(x)	g(x)
-10	$-\frac{1}{10}$
-2	$-\frac{1}{2}$
$-\frac{1}{2}$	-2
$-\frac{1}{10}$	-10
$\frac{1}{10}$	10
$\frac{1}{2}$	2
2	$\frac{1}{2}$
10	$\frac{1}{10}$

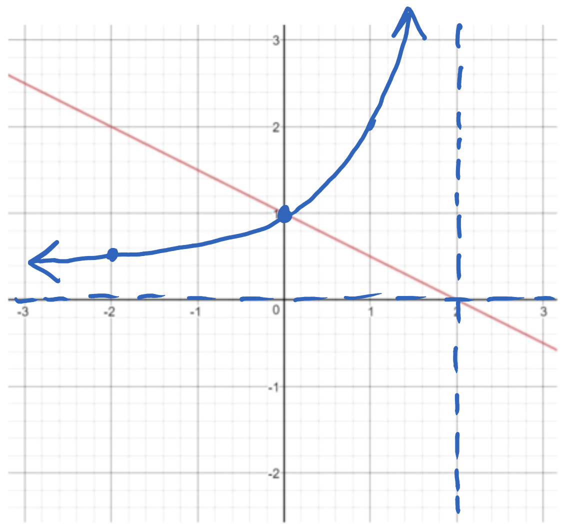
d. On your reciprocal graph, you should now have your horizontal and vertical asymptotes and the invariant points. In order to determine the shape of the rest of the graph, you need to consider the y-coordinates of the original graph.

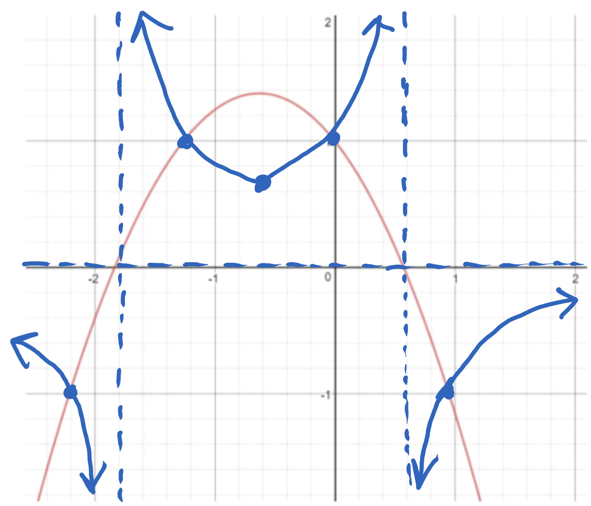
As your y-values become smaller (closer to zero) the reciprocal y-values will become ___larger___.

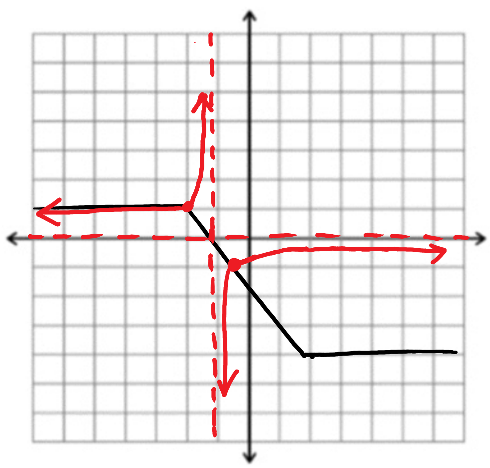
As your y-values become larger (farther from zero) the reciprocal y-values will become ___smaller___.

Let's go back and complete our reciprocal graph!

Example 1: Given the graph of $y = f(x)$ draw the graph of $y = \frac{1}{f(x)}$.







Example 2: State the domain and range of each reciprocal function.

a) $f(x) = \frac{1}{x+4}$

b) $f(x) = \frac{1}{x^2-9}$

Example 3: For the functions $y_1 = 2x$ and $y_2 = \frac{1}{2x}$ use your calculator to

a) Complete the table of values.

x	$y_1 = 2x$	$y_2 = \dfrac{1}{2x}$
-4		
-2		
-1		
$-\dfrac{1}{2}$		
$-\dfrac{1}{4}$		
0		
$\dfrac{1}{4}$		
$\dfrac{1}{2}$		
1		
2		
4		

b) Is the vertical asymptote identified in your table of values? Explain.

The equation of the vertical asymptotes is ________________.

c) Is the horizontal asymptote identified in your table of values? Explain.

d) The equation of the horizontal asymptote is ________________.

e) Write the domain and range for $y_2 = \frac{1}{2x}$.

f) Identify the invariant points:____________________________________.