

# PC11 Lesson 7.3

Tuesday, May 9, 2017 12:12 PM

### 7.3 Absolute Value Equations

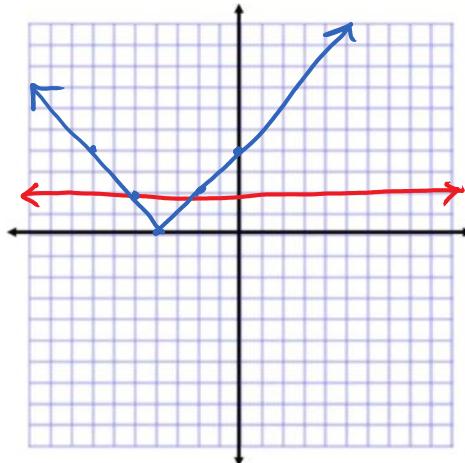
Warm up 1:

- a) Graph  $y = |x + 4|$  and  $y = 2$  using a different colour for each equation.

$$y = |x + 4| \quad y = 2$$

- b) At what points do the graphs intersect?

$$(-6, 2) \text{ and } (-2, 2)$$



- c) Use your calculator to compare the table of values  $y_1 = |x + 4|$  and  $y_2 = 2$ . Complete the table below.

$y =$  math >num l:absc

- d) Does your table of values show your point(s) of intersection? yes

Explain how you could use a table of values to determine the point(s) of intersection.

Look for x-values in which  $y_1 = y_2$

x	$y_1$	$y_2$
-7	3	2
-6	2	2
-5	1	2
-4	0	2
-3	1	2
-2	2	2
-1	3	2

The solution(s) to an absolute value equation, is represented by the x-coordinate of the intersection point(s).

- e) Write  $y_1 = |x + 4|$  in piecewise notation.

$$|x+4| = \begin{cases} -x-4, & x < -4 \\ x+4, & x \geq -4 \end{cases}$$

- f) Solve the equation  $|x + 4| = 2$  algebraically to verify the two solutions calculated in part

(b)?

Case 1  
 $x+4 = 2$   
 $x = -2$

Case 2  
 $-x-4 = 2$   
 $-x = 6$   
 $x = -6$

- g) How do you use your piecewise notation to begin solving an absolute value equation?

We want to solve the eqn for each "piece"

Example 1: Solve each equation algebraically. Verify your solution(s) algebraically.

a)  $|3x - 1| = 4$

solve {

Case 1

$$3x - 1 = 4$$

$$3x = 5$$

$$x = \frac{5}{3} \quad \checkmark$$

verify {

$$\left| 3\left(\frac{5}{3}\right) - 1 \right| = 4 ?$$

$$\left| 5 - 1 \right| = 4$$

$$\left| 4 \right| = 4 \quad \checkmark$$

$$4 = 4$$

Case 2

$$-3x + 1 = 4$$

$$-3x = 3$$

$$x = -1 \quad \checkmark$$

$$\begin{aligned} \left| 3(-1) - 1 \right| &= 4 \\ \left| -3 - 1 \right| &= 4 \\ \left| -4 \right| &= 4 \\ 4 &= 4 \quad \checkmark \end{aligned}$$

$$\therefore x = \frac{5}{3}, -1$$

b)  $|1 - 4x| = 6x$

Case 1

$$1 - 4x = 6x$$

$$1 = 10x$$

$$\frac{1}{10} = x \quad \checkmark$$

$$\left| 1 - 4\left(\frac{1}{10}\right) \right| = 6\left(\frac{1}{10}\right)$$

$$\left| 1 - \frac{2}{5} \right| = \frac{3}{5}$$

$$\left| \frac{3}{5} \right| = \frac{3}{5} \quad \checkmark$$

$$\frac{3}{5} = \frac{3}{5}$$

Case 2

$$-1 + 4x = 6x$$

$$-1 = 2x$$

$$-\frac{1}{2} = x \quad \leftarrow \text{extraneous answer}$$

$$\left| 1 - 4\left(-\frac{1}{2}\right) \right| = 6\left(-\frac{1}{2}\right)$$

$$\left| 1 + 2 \right| = -3$$

$$\left| 3 \right| = -3$$

$$3 = -3 \quad \times$$

$$\therefore x = \frac{1}{10}$$

c)  $|x^2 + 10x + 15| = 6$

Case 1

$$x^2 + 10x + 15 = 6$$

$$x^2 + 10x + 9 = 0$$

$$(x+9)(x+1) = 0$$

$$x+9=0 \quad x+1=0$$

$$x=-9 \quad x=-1$$

verify

$$|(-9)^2 + 10(-9) + 15| = 6 \quad \checkmark$$

$$|(-1)^2 + 10(-1) + 15| = 6$$

$$\begin{aligned} |6| &= 6 \\ 6 &= 6 \quad \checkmark \end{aligned}$$

Case 2

$$-x^2 - 10x - 15 = 6$$

$$0 = x^2 + 10x + 15 + 6$$

$$0 = x^2 + 10x + 21$$

$$0 = (x+3)(x+7)$$

$$\begin{aligned} x+3 &= 0 & x+7 &= 0 \\ x &= -3 & x &= -7 \end{aligned}$$

$$|(-7)^2 + 10(-7) + 15| = 6 \quad \checkmark$$

$$|(-3)^2 + 10(-3) + 15| = 6 \quad \checkmark$$

$$\boxed{\therefore x = -9, -7, -3, -1}$$

d)  $|x^2 + 8x + 16| = 1$

Case 1

$$x^2 + 8x + 16 = 1$$

$$x^2 + 8x + 15 = 0$$

$$(x+3)(x+5) = 0$$

$$x = -3 \quad x = -5$$

verify

$$|(-3)^2 + 8(-3) + 16| = 1$$

$$|9 - 24 + 16| = 1$$

$$|1| = 1 \quad \checkmark$$

Case 2

$$-x^2 - 8x - 16 = 1$$

$$0 = x^2 + 8x + 17$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(17)}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{-4}}{2} \quad \leftarrow \text{No soln}$$

$$|(-5)^2 + 8(-5) + 16| = 1$$

$$|25 - 40 + 16| = 1$$

$$|1| = 1 \quad \checkmark$$

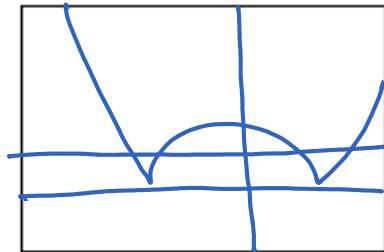
$$\boxed{\therefore x = -3, -5}$$

Example 2: Use your graphing calculator to solve each absolute value equation. Round solutions to two decimal places where necessary.

a)  $|x^2 - 2x - 6| = 4$

$y_1: |x^2 - 2x - 6|$

$y_2: 4$



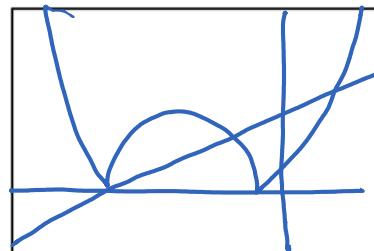
2nd-calc  
5: intersect

Solution(s):  $(-2.32, 4), (-0.73, 4), (2.73, 4), (4.32, 4)$

b)  $3x + 18 = 2|x^2 + 6x|$

$y_1: 3x + 18$

$y_2: 2|x^2 + 6x|$



$x \text{ min: } -10$   
 $\text{max: } 5$   
 $y \text{ min: } -2$   
 $\text{max: } 30$

Solution(s):

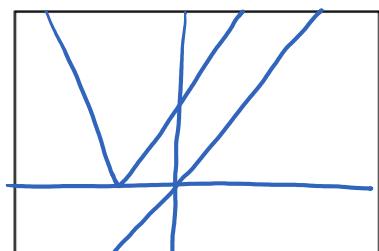
$x = 1.5, -1.5, -6$

$(-6, 0), (-1.5, 13.5), (1.5, 22.5)$

c)  $|2x + 1| = x$

$y_1: |2x + 1|$

$y_2: x$



$x \text{ min: } -5$   
 $\text{max: } 5$   
 $y \text{ min: } -2$   
 $\text{max: } 5$

Solution(s):

No so 1 $\Delta$

Example 3: A manufacturer rejects 250 g boxes of Goldfish Crackers when the actual mass of the box differs from the stated mass by more than 3.5 g.

- a) Write an absolute value equation that can be used to determine the greatest and least masses that are acceptable.

Let  $x$  = weight of box

$$|x - 250| = 3.5$$

- b) Solve the equation. What is the least mass that is acceptable? What is the greatest mass that is acceptable?

$$x - 250 = 3.5$$

$$x = 253.5$$

↑

greatest

$$-x + 250 = 3.5$$

$$x = 246.5$$

↑

least

$$246.5g \leq x \leq 253.5g$$