PC11 Lesson 7.2

Tuesday, May 9, 2017 12:12 PM

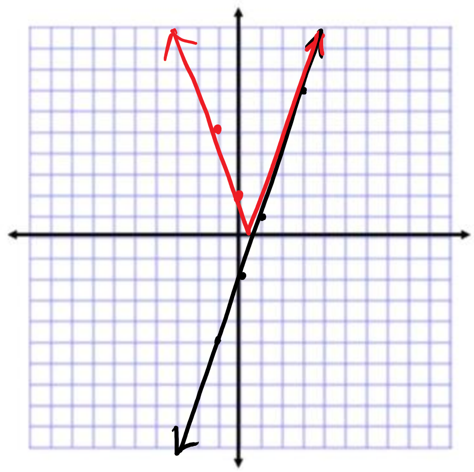
7.2 Absolute Value Functions

Warm up 1:

a) Graph $f(x) = 3x - 2$ and $g(x) = |3x - 2|$ on the same coordinate grid using different colors for each function.

$y = 3x - 2$

$y = |3x-2|$



b) Identify the following characteristics for g(x).

$0 = |3x-2|$

$0 = 3x - 2$

$2 = 3x$

$\frac{2}{3} = x$

x-intercept(s)	$\left(\frac{2}{3}, 0\right)$	
y-intercept	$(0, 2)$	
Domain	$x \in \mathbb{R}$	$(-\infty, \infty)$
Range	$y \geq 0$	$[0, \infty)$

There is a critical point to an absolute value graph where the graph seems to change shape. These points are extremely important when you write the absolute value function in piecewise notation.

Consider the graph in Warm up 1.

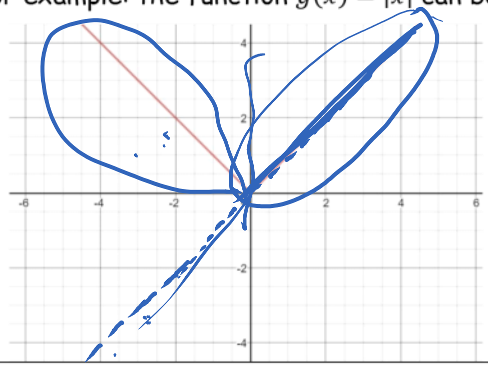
At what point did the graph seem to change its shape? when $x = \frac{2}{3}$

We know this point as an invariant point.

A **piecewise function** is made up of "pieces" of different functions. Each "piece" of the function has its own specific domain.

We use piecewise notation to describe a function that has different definition for different domains. The absolute value of a number is often defined using piecewise notation.

For example: the function $g(x) = |x|$ can be expressed in piecewise notation as:



$f(x)=x$

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

*x-int has to be included only once.

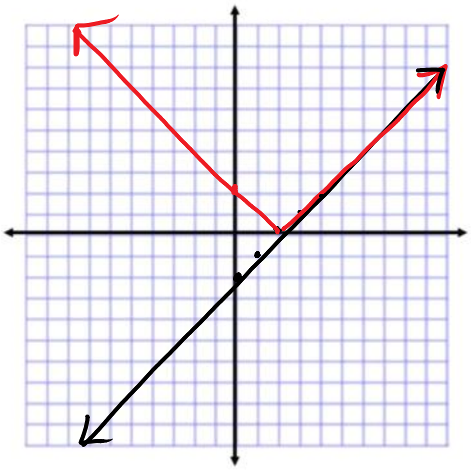
How do we use this critical points to express an absolute value function in piecewise notation?

Each x-intercept will be part of the piecewise notation.

Express $g(x) = |3x - 2|$ in piecewise notation.

$$|3x-2| = \begin{cases} 3x-2, & x \geq \frac{2}{3} \\ -3x+2, & x < \frac{2}{3} \end{cases}$$

Example 1: Graph $y = |x - 2|$ and write the function in piecewise notation.



$y = x - 2$

$y = |x - 2|$

$$|x-2| = \begin{cases} x-2, & x \geq 2 \\ -x+2, & x < 2 \end{cases}$$

Example 2: Express each absolute value function in piecewise notation.

a) $f(x) = |2x + 1|$

x-int:

$0 = 2x + 1$

$-1 = 2x$

$-\frac{1}{2} = x$

$$|2x+1| = \begin{cases} 2x+1, & x \geq -\frac{1}{2} \\ -2x-1, & x < -\frac{1}{2} \end{cases}$$

b) $g(x) = |4 - x|$

x-int:

$0 = 4 - x$

$x = 4$

$$|4-x| = \begin{cases} 4-x, & x \leq 4 \\ -4+x, & x > 4 \end{cases}$$

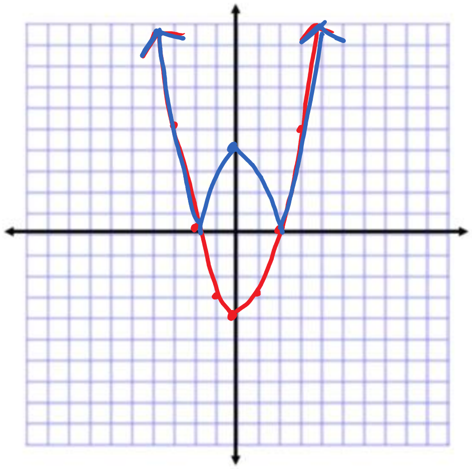
Do you need the graph to write a function in piecewise notation? Explain.

No, it's helpful.

Absolute Value Investigation – Quadratic Relations

1. Complete the table of value for the function $f(x) = x^2 - 4$.
2. Use the table of value to graph $f(x)$.

x	$f(x)$
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5



x	$g(x)$
-3	5
-2	0
-1	3
0	4
1	3
2	0
3	5

3. Complete the table of value for the function $g(x) = |x^2 - 4|$.
4. Use the table of value to graph $g(x)$ in your favourite color.
5. Compare the graphs if f(x) and g(x). What characteristics are
   a. Similar? on the positive y-axis
   b. Different? on the negative y-axis
6. What point(s) is/are the same on the graphs of f(x) and g(x)?
   x-intercepts

   These points are known as invariant points.
7. Express $g(x) = |x^2 - 4|$ in piecewise notation.

$$|x^2-4| = \begin{cases} x^2-4, & x \geq 2 \\ -x^2+4, & -2 < x < 2 \\ x^2-4, & x \leq -2 \end{cases}$$

OR $\begin{cases} x^2-4, & x \leq -2, x \geq 2 \\ -x^2+4, & -2 < x < 2 \end{cases}$

Example 2: For $f(x) = |x^2 - 2x - 8|$

a) Algebraically determine the vertex of $y = x^2 - 2x - 8$.

$= (x^2 - 2x + 1 - 1) - 8$

$= (x^2 - 2x + 1) - 9$

$= (x - 1)^2 - 9$ vertex $(1, -9)$

b) Algebraically determine the x and y-intercepts of $y = x^2 - 2x - 8$

$0 = x^2 - 2x - 8$

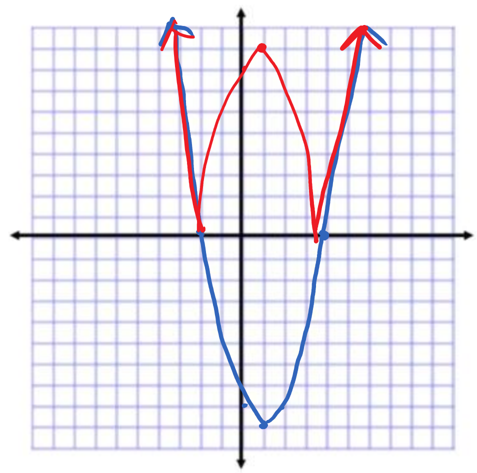
$0 = (x - 4)(x + 2)$

$x = 4 \quad x = -2$

$y = 0^2 - 2(0) - 8$

$y = -8$

c) Graph $y = x^2 - 2x - 8$ and $f(x) = |x^2 - 2x - 8|$ on the same coordinate grid.



d) Identify the domain and range of $f(x) = |x^2 - 2x - 8|$.

Domain: $x \in \mathbb{R}$

Range: $y \geq 0$

e) Write $f(x) = |x^2 - 2x - 8|$ in piecewise notation.

$$|x^2 - 2x - 8| = \begin{cases} x^2 - 2x - 8, & x \geq 4 \\ -x^2 + 2x + 8, & -2 < x < 4 \\ x^2 - 2x - 8, & x \leq -2 \end{cases}$$