PC11 Lesson 6.4

Wednesday, April 26, 2017 4:41 PM

6.4 Rational Equations

To solve equations with rational expressions:

1. factor if possible
2. identify NPV of the variables
3. multiply both sides of the equation by common denominator
4. solve for the variables.
5. check your answer.

Example 1: Solve each equation:

a. $\left(\frac{x}{5}\right)=\left(\frac{2}{x}+\frac{(x+3)}{5}\right)5x$

LCD: $5x$

NPV $x\neq 0$

$\frac{5x^2}{5}=\frac{10x}{x}+\frac{5x(x+3)}{5}$

$x^2=10+x(x+3)$

$x^2=10+x^2+3x$

$0=x^2-x^2+3x+10$

$0=3x+10$

$-10=3x$

$-\frac{10}{3}=x$

Check

$-\frac{10}{3}\div 5=\frac{2}{(-10/3)}+\frac{-10/3+3}{5}$

$-\frac{10}{15}=\frac{6}{-10}-\frac{1}{15}$

$-\frac{10}{15}=-\frac{10}{15}$ ✓

b. $x^2\left(3+\frac{1}{x}\right)=\left(\frac{4}{x^2}\right)x^2$

LCD: x^2

NPV: $x\neq 0$

$3x^2+\frac{x^2}{x}=\frac{4x^2}{x^2}$

$3x^2+x=4$

$3x^2+x-4=0$

$(3x+4)(x-1)=0$

$x=-\frac{4}{3},\ 1$

Check

$3+\frac{1}{-4/3}=\frac{4}{\left(-\frac{4}{3}\right)^2}$ ✓

$3+\frac{1}{1}=\frac{4}{1^2}$

$3+1=4$ ✓

Method 2 Match denominators

Example 2: Solve each equation:

a. $\frac{5}{x-2}=\frac{9}{3x-6}$

NPV: $x \neq 2$

$\frac{5}{x-2}=\frac{9}{3(x-2)}$

$(x-2)\frac{5}{x-2}=\frac{3}{x-2}(x-2)$

$5 \neq 3$

$\therefore$ No Soln

Method 3 Cross multiply

c. $\frac{3x+1}{x^2-1}=\frac{-x}{x+1}$

NPV

$x = -1, 1$

$(3x+1)(x+1) = -x(x^2-1)$

$\frac{(3x+1)(x+1)}{x+1} = \frac{-x(x+1)(x-1)}{x+1}$

$3x+1 = -x(x-1)$

$3x+1 = -x^2+x$

$x^2+2x+1=0$

$(x+1)(x+1)=0$

$x=-1$

$\therefore$ No Soln

Method 2

Example 3: Solve each equation:

a. $\frac{x}{x-3}=\frac{-6}{x^2-8x+15}$

$\frac{(x-5)}{(x-5)}\frac{x}{x-3}=\frac{-6}{(x-3)(x-5)}$

$\frac{x(x-5)}{(x-3)(x-5)}=\frac{-6}{(x-3)(x-5)}$

$x(x-5) = -6$

NPV

$x^2-5x+6=0$

$x \neq 3,5$

$(x-2)(x-3)=0$

$x=2, 3$ reject

$\therefore x=2$

b. $\frac{x}{x+1}-\frac{x+1}{x-4}=\frac{5}{x^2-3x-4}$

$\frac{(x-4)}{(x-4)}\cdot\frac{x}{(x+1)}-\frac{(x+1)}{(x-4)}\frac{(x+1)}{(x+1)}=\frac{5}{(x+1)(x-4)}$

$\frac{x(x-4)-(x+1)(x+1)}{(x+1)(x-4)}=\frac{5}{(x+1)(x-4)}$

$x(x-4)-(x+1)(x+1)=5$

$x^2-4x-(x^2+2x+1)=5$

$x^2-4x-x^2-2x-1=5$

$-6x-1=5$

$-6x=6$

$x=-1$

$\therefore$ No Soln

NPV

$x \neq -1, 4$

Rational equations can be used to solve a variety of real-world problems, including those involving motion, work, and proportions.

$\text{Speed} = \frac{\text{Distance}}{\text{time}} \Rightarrow \text{time} = \frac{\text{Distance}}{\text{speed}}$.

Example 1: A boat travels at an average speed of 15 km/h in still water. The boat travels 12 km downstream in the same time as it travels 8 km upstream. Determine the average speed of the current.

Let x = speed of current

	Distance	Speed	Time
Downstream	12	$15+x$	$\frac{12}{15+x}$
Upstream	8	$15-x$	$\frac{8}{15-x}$

$$(15+x)(15-x)\left(\frac{12}{15+x}\right) = \left(\frac{8}{15-x}\right)(15+x)(15-x)$$

$$\frac{12(15+x)(15-x)}{15+x} = \frac{8(15+x)(15-x)}{15-x}$$

$$12(15-x) = 8(15+x)$$

$$180 - 12x = 120 + 8x$$

$$60 = 20x$$

$$3 = x$$

The average speed of the current is 3 km/h.

Example 2: Kyra mows a lawn in 40 min. When Mark and Kyra work together, they can mow the lawn in 24 min. How long would it take Mark to mow the lawn on his own?

Let x = time it took Mark by himself.

	Time	Fraction of work done in 1 min.	Fraction of work done in 24 min.
Kyra	40	$\frac{1}{40}$	$24 \cdot \frac{1}{40} = \frac{24}{40}$
Mark	x	$\frac{1}{x}$	$\frac{24}{x}$
Together	24	$\frac{1}{24}$	$\frac{24}{24} = 1$

$$40x\left(\frac{24}{40} + \frac{24}{x}\right) = (1)\,40x$$

$$24 + \frac{960x}{40} + \frac{960x}{x} = 40x$$

$$24x + 960 = 40x$$

$$960 = 16x$$

$$60 = x$$

On his own it takes Mark 60 min.

Example 3: How much lemon juice must be added to 2 L of water to make a lemonade solution that contain 20% lemon juice?