

PC11 Lesson 6.3

Wednesday, April 26, 2017 4:40 PM

6.3 Adding and Subtracting Rational Expressions

Warm Up

Common denominator to add/subtract fractions

Add or Subtract the following:

$$\begin{array}{llll}
 \text{a) } \frac{4}{5} + \frac{3}{5} & \text{b) } \frac{3}{7} + \frac{3}{14} & \text{c) } \frac{1}{6} - \frac{2}{9} & \text{d) } \frac{21}{8} - \frac{11}{3} \\
 = \frac{7}{5} & = \frac{6}{14} + \frac{3}{14} & = \frac{3}{18} - \frac{4}{18} & = \frac{63}{24} - \frac{88}{24} = -\frac{25}{24} \\
 & = \frac{9}{14} & = -\frac{1}{18} &
 \end{array}$$

Rational expressions are added and subtracted the same way that fractions are. You must have a **common denominator** in order to combine rational expressions. You can use any equivalent common denominator, but it is usually easier to use the **lowest common denominator (LCD)**.

Example 1: Simplify:

$$\begin{aligned}
 \text{a. } \frac{4}{3x} - \frac{7x}{6} &\quad \text{LCD: } 6x \quad \text{NPV} \quad \text{d. } \frac{2a+1}{2a^2b} - \frac{b-3}{9ab^2} \quad \text{LCD: } 18a^2b^2 \quad \text{NPV} \\
 &= \frac{4}{3x} - \frac{7x}{6} \\
 &= \frac{8}{6x} - \frac{7x^2}{6x} \\
 &= \frac{8-7x^2}{6x} \\
 &= \frac{(2a+1)(9b)}{(2a^2b)(9b)} - \frac{(b-3)(2a)}{9ab^2(2a)} \quad a \neq 0 \quad b \neq 0 \\
 &= \frac{18ab+9b}{18a^2b^2} - \frac{2ab-6a}{18a^2b^2} \\
 &= \frac{18ab+9b-(2ab-6a)}{18a^2b^2} \\
 &= \frac{18ab+9b-2ab+6a}{18a^2b^2} \\
 &= \frac{16ab+9b+6a}{18a^2b^2}
 \end{aligned}$$

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Example 2: Simplify:

LCD: $m-2$

$$\text{a. } \frac{m-4}{m-2} - \frac{m-10}{m-2}$$

NPV

$m \neq 2$

$$= \frac{m-4-(m-10)}{m-2}$$

$$= \frac{m-4-m+10}{m-2}$$

$$= \frac{6}{m-2}$$

$$\text{d. } \frac{5}{x-1} - \frac{12}{x^2-1}$$

LCD: x^2-1

$$\text{NPV } x \neq \pm 1 \quad = \frac{5(x+1)}{(x-1)(x+1)} - \frac{12}{x^2-1}$$

$$= \frac{5x+5-12}{x^2-1}$$

$$= \frac{5x-7}{x^2-1}$$

$$\text{b. } \frac{5}{x} + \frac{6x}{x+4}$$

$$= \frac{5(x+4)}{x(x+4)} + \frac{6x(x)}{(x+4)(x)}$$

$$= \frac{5x+20+6x^2}{x(x+4)}$$

LCD: $x(x+4)$

NPV

$x \neq 0, -4$

$$\text{f. } \frac{x}{x^2-9x+18} - \frac{x-2}{x^2-10x+24}$$

Factor 1st

$$= \frac{x}{(x-6)(x-3)} - \frac{x-2}{(x-6)(x-4)}$$

LCD: $(x-6)(x-4)(x-3)$

NPV $x \neq 6, 4, 3$

$$= \frac{x(x-4)}{(x-6)(x-3)(x-4)} - \frac{(x-2)(x-3)}{(x-6)(x-3)(x-4)}$$

$$= \frac{x^2-4x - (x^2-5x+6)}{(x-6)(x-3)(x-4)}$$

$$= \frac{x-6}{(x-6)(x-3)(x-4)}$$

$$= \frac{1}{(x-3)(x-4)}$$

Sometimes we have a fraction within a fraction. Eg. $\frac{2+\frac{3}{4}}{5-\frac{1}{3}}$.

This is called a **complex fraction**. To simplify a complex fraction, we must multiply the numerator and denominator by the common denominator of the internal fractions.

Example 3: Simplify:

$$\begin{array}{ll}
 \text{a. } \frac{\left(\frac{1}{2}+x\right)6}{\left(\frac{1}{3}-x\right)6} & \text{b. } \frac{\left(\frac{1}{x}+2\right)x}{\left(\frac{1}{x}-3\right)x} \\
 \text{LCD: 6} & \text{LCD: } x \\
 \text{NPV } x \neq \frac{1}{3} & \text{NPV } x \neq \frac{1}{3} \\
 & = \frac{3(1+2x)}{2(1-3x)} \\
 & = \frac{6}{2} + 6x \\
 & = \frac{3+6x}{2-6x} \\
 & = \frac{3(1+2x)}{2(1-3x)} \\
 & = \frac{x}{x} + 2x \\
 & = \frac{x}{x} - 3x \\
 & = \frac{1+2x}{1-3x}
 \end{array}$$

$$\begin{array}{l}
 \text{c. } \frac{\left(2-\frac{4}{y}\right)y}{\left(y-\frac{4}{y}\right)y} \\
 \text{LCD: } y \\
 y \neq \pm 2 \\
 = \frac{2y-4}{y^2-4} = \frac{2(y-2)}{(y+2)(y-2)} = \frac{2}{y+2}
 \end{array}$$

$$\begin{array}{l}
 y-\frac{4}{y} \neq 0 \\
 y \neq \frac{4}{y} \\
 y^2 \neq 4
 \end{array}$$