

PC11 Lesson 5.3

Monday, April 17, 2017 8:53 PM

5.3 Radical Equations (Restrictions)

Warm Up:

An **inequality** is a mathematical statement comparing expressions that may not be equal. These can be written using the symbols less than ($<$), greater than ($>$), less than or equal to (\leq), greater than or equal to (\geq), and not equal to (\neq).

Example 1: Place a less than symbol ($<$), or a greater than symbol ($>$), in the box.

a) $7 \boxed{<} 15$

b) $579 \boxed{>} 246$

c) $-18 \boxed{<} -16$

Example 2: Solve for x .

a) $3x + 12 = 27$

$$\begin{aligned} 3x &= 15 \\ x &= 5 \end{aligned}$$

b) $\frac{2}{3}x - 5 = -7$

$$x = -3$$

$$\begin{aligned} c) 5x - 2 &= 7x - 5 \\ +5 &\quad +5 \\ 5x + 3 &= 7x \\ -5x &\quad -5x \\ 3 &= 2x \\ \frac{3}{2} &= x \end{aligned}$$

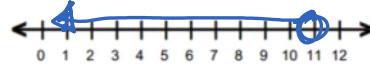
Example 3: Solve the following inequalities. Represent your answer on a number line.

a) $14 \leq 7 + a$

$$7 \leq a$$

b) $9 > -2 + x$

$$x < 11$$



Example 4: Identify any restrictions on the variable

a) $\sqrt{14x}$

$$14x \geq 0$$

$$x \geq 0$$

b) $\sqrt{15x + 25} \geq 0$

$$15x + 25 \geq 0$$

$$15x \geq -25$$

$$x \geq -\frac{25}{15} \text{ or } -\frac{5}{3}$$

$$x \geq -1.6$$

c) $\sqrt[6]{2(4x - 13)} + 5$

$$2(4x - 13) \geq 0$$

$$4x - 13 \geq 0$$

$$4x \geq 13$$

$$x \geq \frac{13}{4} \text{ or } 3.25$$

d) $\sqrt{5 - 3x} + 7 = \sqrt{8 - 2x}$

$$5 - 3x \geq 0 \text{ or } 5 - 3x \geq 0 \quad \left. \begin{array}{l} 8 - 2x \geq 0 \\ 8 \geq 2x \end{array} \right\}$$

$$-3x \geq -5$$

$$5 \geq 3x$$

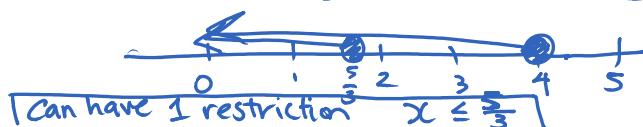
$$x \leq \frac{5}{3}$$

$$\frac{5}{3} \geq x$$

$$8 \geq 2x$$

$$4 \geq x$$

$$x \leq 4$$



A radical equation is an equation with at least one radical whose radicand contains a variable. A solution to a radical equation is a root of the equation.

When solving a radical equation, remember to:

1. Identify any restrictions of the variable
2. Identify whether any roots are extraneous by determining whether the values satisfy the original equation.

Example 1: Solve each equation and verify the solution:

| | |
|---|--|
| <p>a. $3\sqrt{x} = 5$</p> <p>Restrictions $x \geq 0$</p> $\frac{3}{(\sqrt{x})^2} = \left(\frac{5}{3}\right)^2$ <p>$x = \frac{25}{9}$</p> <p>Within the restrictions ✓</p> <p>Verify $\sqrt{\frac{25}{9}} = 5$</p> $\frac{3\sqrt{\frac{25}{9}}}{3 \times \frac{5}{3}} = 5$ | <p>b. $4\sqrt{x+1} - 5 = 3$</p> <p>Restrictions $x+1 \geq 0$ $x \geq -1$</p> <p>$4\sqrt{x+1} - 5 = 3$</p> $4\sqrt{x+1} = 8$ $(\sqrt{x+1})^2 = (2)^2$ <p>$x+1 = 4$ $x = 3$</p> <p>within the restriction ✓</p> <p>Verify $\sqrt{3+1} - 5 = 3$</p> $4(2) - 5 = 3$ $3 = 3$ |
|---|--|

Example 2: Solve for x : $\sqrt{3x+3} - x = 1$

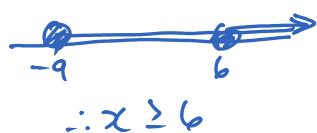
| | |
|--|--|
| <p>Restrictions $3x+3 \geq 0$ $3x \geq -3$ $x \geq -1$</p> $(\sqrt{3x+3})^2 = (1+x)^2$ $3x+3 = (1+x)(1+x)$ $3x+3 = 1 + 2x + x^2$ $3x+3 = x^2 + 2x + 1$ $0 = x^2 - x - 2$ $0 = (x-2)(x+1)$ $x-2=0 \quad x+1=0$ $x=2 \quad x=-1$ <p>Within the restrictions ✓</p> | <p>Verify $x=2$</p> $\sqrt{3(2)+3} - 2 = 1$ $\sqrt{9} - 2 = 1$ $3 - 2 = 1 \checkmark$ <hr/> <p>Verify $x=-1$</p> $\sqrt{3(-1)+3} - (-1) = 1$ $\sqrt{-3+3} + 1 = 1$ $\sqrt{0} + 1 = 1$ $1 = 1 \checkmark$ |
|--|--|

Example 4: Solve for x : $\sqrt{x+9} - \sqrt{x-6} = 3$

Restrictions

$$x+9 \geq 0 \quad x-6 \geq 0$$

$$x \geq -9 \quad x \geq 6$$



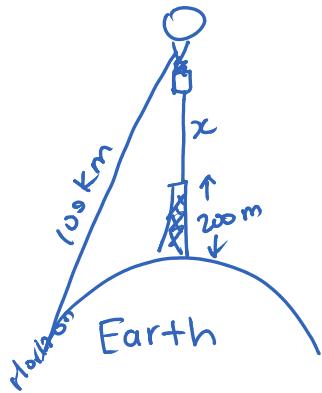
$$\therefore x \geq 6$$

$$\begin{aligned}
 (\sqrt{x+9})^2 &= (\sqrt{x-6} + 3)^2 \\
 x+9 &= (\underbrace{\sqrt{x-6} + 3}_{\text{Verify}})(\underbrace{\sqrt{x-6} + 3}_{\text{Verify}}) \\
 x+9 &= (x-6) + 3\sqrt{x-6} + 3\sqrt{x-6} + 9 \\
 x+9 &= x-6 + 6\sqrt{x-6} + 9 \\
 x+9 &= x+3+6\sqrt{x-6} \\
 6 &= 6\sqrt{x-6} \\
 (6)^2 &= (\sqrt{x-6})^2 \\
 36 &= x-6 \\
 42 &= x
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{7+9} - \sqrt{7-6} &= 3 \\
 \sqrt{16} - \sqrt{1} &= 3 \\
 4 - 1 &= 3 \checkmark
 \end{aligned}$$

Example 5: The formula $d = \sqrt{13h}$ can be used to estimate the distance to the horizon, d km, from an observer's height, h m, above sea level. An observer is in a hot air balloon that is attached to the top of a 200 m tower whose base is at sea level. How high above the tower must the balloon be so the observer's distance to the horizon is 100 km?

$$\sqrt{d} = 100$$



Restrictions

$$13(200+x) \geq 0$$

$$200+x \geq 0$$

$$\begin{aligned}
 100 &= \sqrt{13h} \\
 (100)^2 &= (\sqrt{13}(200+x))^2 \\
 10000 &= 13(200+x)
 \end{aligned}$$

$$10000 = 2600 + 13x$$

$$7400 = 13x$$

$$569.23\dots = x$$

The balloon must be
569.23 m above
the tower

Homework: p. 300 # 1, 3-10 (a,c), 11, 16-18, 23