PC11 Lesson 5.1

Saturday, February 4, 2017 4:45 PM

Ch.5 Note package

5.1: Working with Radicals

Warm Up:

Fill in the blanks using numerical values.

a) 6×6 is equal to __36__, so 6 is a square root of __36__.

b) The square of 9 is equal to __81__, so 9 is a square root of __81__.

c) 8^2 is equal to __64__, so __64__ is the square of __8__ and __8__ is a square root of __64__.

d) 5×5×5 is equal to __125__, so 5 is a cube root of __125__.

e) 2^3 is equal to __8__, so __8__ is the cube of __2__ and __2__ is a cube root of __8__.

Which of the following numbers are perfect squares, perfect cubes, or both?

a) 144 perfect square   b) 2196 Neither   c) 16 perfect square   d) 64 Both

$4^3 = 64$ $8^2 = 64$

Remember your exponent rules? Simplify the following. (Leave only positive exponents)

a) $(x^3)\left(x^{\frac{2}{5}}\right)$

$= x^{3+\frac{2}{5}}$

$= x^{\frac{17}{5}}$ or $\sqrt[5]{x^{17}}$

b) $\dfrac{(m^{-2})^{\frac{2}{3}}}{\left(m^{\frac{1}{2}}\right)^5} = \dfrac{m^{-\frac{4}{3}}}{m^{\frac{5}{2}}}$

$= \dfrac{1}{m^{4/3}m^{5/2}}$

$= \dfrac{1}{m^{23/6}}$

c) $(d^4)^{-\frac{3}{8}}$

$= d^{-\frac{3}{2}}$

$= \dfrac{1}{d^{3/2}}$

d) $\left[\dfrac{(x^{-2})}{(xy)^3}\right]^{1.5} = \dfrac{x^{-3}}{x^{4.5}y^{4.5}}$

$= \dfrac{1}{x^{7.5}y^{4.5}}$ or $\dfrac{1}{x^{\frac{15}{2}}y^{\frac{9}{2}}}$

Evaluate (without a calculator)

a) $[(5)(5^3)]^{-1}$

$= 5^{-1}5^{-3}$

$= 5^{-4} = \dfrac{1}{625}$

b) $\left(3^{\frac{8}{3}}\right)\left(3^{\frac{1}{3}}\right)$

$= 3^{\frac{9}{3}}$

$= 3^3 = 27$

c) $\left[\left(\frac{3}{4}\right)^{-4} \div \left(\frac{3}{4}\right)^2\right]^{-1}$

$\left[\dfrac{(3/4)^{-4}}{(3/4)^2}\right]^{-1}$

$= \dfrac{(3/4)^4}{(3/4)^{-2}} = \left(\frac{3}{4}\right)^4\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)^6 = \dfrac{729}{4096}$

d) $\left(\dfrac{2^3}{8^2}\right)^{\frac{4}{3}} = \dfrac{2^4}{8^{8/3}} = \dfrac{16}{256} = \dfrac{1}{16}$

Simply by combining like terms.

a) $3x - 7y^2 - 4x + 9y$ **b)** $3am - 2pm + am - 4pm$ **c)** $3x^2 - 4x^2 + 5x^2y$ **d)** $p^3 + q^2y - p^3 + qy^2 + 5q^2y$

$= -x - 7y^2 + 9y$ $= 4am - 6pm$ $= -x^2 + 5x^2y$ $= 6q^2y + qy^2$

"liketerms" have the exact same variable combination

Radicals

Radical

Index or Root $\sqrt[3]{64}$ Radicand

There are two types of radicals:

Entire radicals: The entire number is the radicand. ex) $\sqrt{20}$

Mixed radicals: Coefficient in front of radical other than 1.

ex) $2\sqrt{5}$

Remember from Grade 10: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and vice versa. Therefore, we can convert between entire and mixed radicals.

Example 1: Express each entire radical as a mixed radical in simplest form.

45 = 5 × 9, 9 = 3 × 3

$$\sqrt{45}$$
$$= \sqrt{3 \times 3 \times 5}$$
$$= 3\sqrt{5}$$

$40x^3y$ = 10 × 4 × x^2 × x × y; 10 = 2 × 5, 4 = 2 × 2, $x^2 = x \times x$

$$\sqrt[3]{40x^3y}$$
$$= \sqrt[3]{2 \times 2 \times 2 \times 5 \times x \times x \times x \times y}$$
$$= 2x\sqrt[3]{5y}$$

** If your index is even, the radicand must be positive. ex) $\sqrt{x}$, $x \geq 0$

Example 2: Express each mixed radical in entire radical form.

$$2\sqrt{3}$$
$$= \sqrt{2 \times 2 \times 3}$$
$$= \sqrt{12}$$

$$-2\sqrt[4]{\frac{3}{4}}$$
$$= -\sqrt[4]{2^4 \cdot \frac{3}{4}} = -\sqrt[4]{16 \cdot \frac{3}{4}}$$
$$= -\sqrt[4]{12}$$

Example 3: Arrange in order from least to greatest

$$9\sqrt{2}, 2\sqrt{6}, 8\sqrt{3}$$

$\sqrt{162}$ $\sqrt{24}$ $\sqrt{192}$

$2\sqrt{6}$, $9\sqrt{2}$, $8\sqrt{3}$

Restrictions on Variables:

If a radical represents a real number and has an **even** index, the radicand must be greater than or equal to 0. Eg. $\sqrt{4-x} \Rightarrow$

Restrictions
$4-x \geq 0$
$4 \geq x$ or $x \leq 4$

If a radical represents a real number and has on **odd** index, the radicand can be any real number.

Example 1: For which values of the variable is each radical defined?

a. $\sqrt{54x^3}$

$54x^3 \geq 0$
$\sqrt[3]{x^3} \geq \sqrt[3]{0}$
$x \geq 0$

b. $\sqrt[3]{12x^5}$

odd index
$x \in \mathbb{R}$

c. $\sqrt[4]{2x-7}$

$2x-7 \geq 0$
$2x \geq 7$
$x \geq \frac{7}{2}$ or $x \geq 3.5$

Like Radicals:

Radicals with the same **radicand** and **index** are called **like radicals**. When adding or subtracting radicals, they must be "like" radicals or they cannot be combined.

Always simplify radicals first

Example 2: Simplify and combine like radicals:

a. $5\sqrt{6}-2\sqrt{6}$

$= 3\sqrt{6}$

b. $\sqrt[3]{128}-\sqrt[3]{16}-\sqrt[3]{54}$

$= 4\sqrt[3]{2} - 2\sqrt[3]{2} - 3\sqrt[3]{2}$

$= -1\sqrt[3]{2} = -\sqrt[3]{2}$

Example 3: Simplify:

a. $2\sqrt{x}-3\sqrt{y}+5\sqrt{x}+2\sqrt{y}$

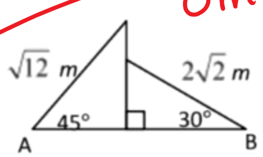
$= 7\sqrt{x} - \sqrt{y}$

b. $8\sqrt[3]{2x}+7\sqrt{2x}-5\sqrt[3]{2x}+\sqrt{2x}$

$= 3\sqrt[3]{2x} + 8\sqrt{2x}$

Example 4: ~~What is the exact length of AB?~~

OMIT



Homework: p. 278 # 1, 4, 8 – 10 (a, c), 15, 20, 23