

Lesson 4.2

Saturday, February 4, 2017 4:41 PM

* Always check for common factors first *

PREC 11

4.2 Factoring Quadratic Equations

Warm-Up:

1. Factor each trinomial.

a) $x^2 + 4x - 21$

$= (x-3)(x+7)$

b) $2x^2 - 7x + 6$

$= (2x-3)(x-2)$

2. Factor each expression fully.

a) $-3x^2 + 9xy - 6y^2$

$= -3(x^2 - 3xy + 2y^2)$

b) $3x^3 - 3x^2 + 27x$

$= 3x(x^2 - x + 9)$

cannot factor further.

c) $x^4 - 16$

$= (x^2 + 4)(x^2 - 4)$

$= (x^2 + 4)(x+2)(x-2)$

d) $(3x+1)^2 - (2x-3)^2$

$= [(3x+1) + (2x-3)][(3x+1) - (2x-3)]$

$= (3x+1+2x-3)(3x+1-2x+3)$

$= (5x-2)(x+4)$

* Use a variable to equal the expression in the brackets *

Example 1: Factor each polynomial:

$$\begin{aligned}
 & \text{Let } m = x+2 \\
 \text{a.} \quad & 12(x+2)^2 + 24(x+2) + 9 \\
 & = 12m^2 + 24m + 9 \\
 & = 3(4m^2 + 8m + 3) \\
 & = 3(2m+3)(2m+1) \\
 & = 3(2(x+2)+3)(2(x+2)+1) \\
 & = 3(2x+4+3)(2x+4+1) \\
 & = 3(2x+7)(2x+5)
 \end{aligned}$$

$$\begin{aligned}
 & \text{Let } p = n+3 \\
 \text{b.} \quad & -2(n+3)^2 + 12(n+3) + 14 \\
 & = -2p^2 + 12p + 14 \\
 & = -2(p^2 - 6p - 7) \\
 & = -2(p+1)(p-7) \\
 & \quad \quad \quad \downarrow \\
 & = -2(n+4)(n-4)
 \end{aligned}$$

Many quadratic equations can be solved by factoring. The zero product property states that if $ab = 0$, then $a = 0$ or $b = 0$, or both. Therefore, the roots of a quadratic equation occur when the product of the factors is equal to zero.

Example 2: Solve each equation, then verify the solution.

a. $x^2 - 2x - 8 = 0$

$$(x+2)(x-4) = 0$$

Equals zero when $x+2=0$

$$\text{OR } x-4=0$$

$$x+2=0$$

$x = -2$

$$x-4=0$$

$x = 4$

b. $3x^2 - 2x - 8 = 0$

$$(3x+4)(x-2) = 0$$



$$3x+4=0$$

$$3x = -4$$

$x = -\frac{4}{3}$

$$x-2=0$$

$x = 2$

c. $2x^2 + 18 = 12x$

$$2x^2 + 18 - 12x = 0$$

$$2x^2 - 12x + 18 = 0$$

$$2(x^2 - 6x + 9) = 0$$

$$2(x-3)(x-3) = 0$$

$$x = 3 \quad x = 3$$

$$\therefore x = 3$$

d. $2x^2 = 4x$

$$2x^2 - 4x = 0$$

$$2x(x-2) = 0$$



$$2x = 0$$

$x = 0$

$$x-2=0$$

$x = 2$

Example 3: A rectangular garden has dimensions 5 m by 7 m. When both dimensions are increased by the same length, the area of the garden increases by 45 m². Determine the dimensions of the larger garden.

Old
7

$$\boxed{A = 35 \text{ m}^2}$$

5

New

$$\begin{aligned} & 7+x \\ & A = 35 + 45 \\ & = 80 \text{ m}^2 \end{aligned}$$

5+x

$$(7+x)(5+x) = 80$$

$$35 + 7x + 5x + x^2 = 80$$

$$x^2 + 12x + 35 - 80 = 0$$

$$x^2 + 12x - 45 = 0$$

$$(x-3)(x+15) = 0$$

$$x-3=0 \quad \text{or} \quad x+15=0$$

$$\begin{array}{c} x=3 \\ \text{---} \end{array}$$

$$\begin{array}{c} x=-15 \\ \text{---} \\ \text{reject} \end{array}$$

∴ New dimensions
are 10m by 8m

Example 4: A football is kicked vertically. The approximate height of the football, h metres, after t seconds is modeled by the formula: $h = 1 + 20t - 5t^2$.

- a. Determine the height of the football after 2 s.

$$\begin{aligned} h &= 1 + 20(2) - 5(2)^2 \\ &= 1 + 40 - 5(4) \\ &= 1 + 40 - 20 \\ &= 21 \text{ m} \end{aligned}$$

- b. When is the football 16 m high?

$$16 = 1 + 20t - 5t^2$$

$$h=16$$

$$5t^2 - 20t + 15 = 0$$

$$5(t^2 - 4t + 3) = 0$$

$$5(t-1)(t-3) = 0$$

$$\therefore t = 1 \text{ or } 3$$

The football is 16m high
at 1 sec and 3 sec.

Assignment: pg. 229 #1-6 (ac), 8ace, 10-13, 15, 28

First, I looked in
the back of the book,
but it wasn't an odd-
numbered problem.



Then I asked my
little brother, but
he wanted me to
pay him \$5.



Finally, I found it
on the Internet
with Google.



MY MATH
TEACHER
WANTS US TO
SHOW HOW WE
GET OUR
ANSWERS.



HW

P.229

4-6 ac
8 ace
10 ace
11-15, 26,
28