Lesson 3.3.2

Saturday, February 4, 2017 4:39 PM

PREC 11 — **3.3 (cont.) Completing the Square to Solve Max or Min Problems**

When a quadratic function is graphed, the vertex is either the highest or lowest point on the curve.

Examples: **1.** $y = 2(x-3)^2 + 4$ y is <u>min</u> at <u>4</u> when <u>$x=3$</u>.

positive

vertex $(3,4)$

2. $y = -3(x-4)^2 + 1$ y is <u>max</u> at <u>1</u> when <u>$x=4$</u>.

negative

3. Find the max or min of $y = -4x^2 - 12x + 5$.

$$y = -4\left(x + \frac{3}{2}\right)^2 + 14$$

negative

vertex: $\left(\frac{3}{2}, 14\right)$

$\therefore$ y in max. at 14 when $x = -\frac{3}{2}$

Example 4: Two numbers have a difference of 10. Their product is a minimum. What are the numbers?

Let x = one #

Let $x+10$ = other #

$P = x(x+10)$

$= x^2 + 10x$

$P = (x+5)^2 - 25$

$x = -5$

$-5 + 10 = 5$

Product is minimum of -25 when $x = -5$.

$\therefore$ The numbers are -5 and 5.

Example 5: Two numbers have a sum of 24. Their product is a maximum. What are the numbers?

Let x = one #

$24 - x$ = other #

$P = x(24-x)$

$P = 24x - x^2$

$P = -x^2 + 24x$

$P = -(x-12)^2 + 144$

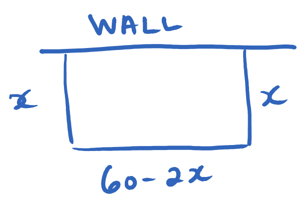
$24 - 12 = 12$

Max. product of 144 when $x = 12$

$\therefore$ The numbers are 12 and 12.

Example 6: A rectangular playground is bounded on one side by a wall and on the other three sides by 60 m of fencing. Determine the dimensions of the largest possible playground.

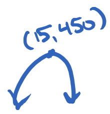
↳ Maximum Area



$$A = x(60-2x)$$
$$= 60x - 2x^2$$
$$= -2x^2 + 60x$$

$$\Downarrow$$

$$A = -2(x-15)^2 + 450$$



Max. Area of 450m^2 when $x = 15$

To find length
$60-2(15)$
$= 30$

$\boxed{\therefore \text{Length} = 30\text{m and width} = 15\text{m}}$

Example 7: A theatre company currently charges \$12 a ticket. At this price 450 people attend each show. For every \$2 increase in price, 25 fewer people will attend the show. What is the maximum revenue?

Let x = # of \$2 increases in price.

Revenue = (# of tickets)(price)

$$= (450-25x)(12+2x)$$
$$= 5400 + 900x - 300x - 50x^2$$
$$= -50x^2 + 600x + 5400$$
$$= -50(x^2 - 12x + 36 - 36) + 5400$$
$$= -50(x-6)^2 + 7200$$

Vertex
(6, 7200)
x is # of \$2 increase.
max. Revenue

The maximum revenue is \$7200.

Assignment: pg. 192 ~~#13,14,16,18-21,23,24~~ 15,16,19,22-24, 25,26,27