Lesson 3.3.1

Saturday, February 4, 2017 4:39 PM

PREC 11 **3.3 Completing the Square**

Review:

The **standard form** of a quadratic function has the equation $y = ax^2 + bx + c$.

The **vertex form** of a quadratic function has the equation $y = a(x-p)^2 + q$.

perfect square

Writing the equation in vertex form enables us to analyze the function more easily as we can determine the vertex, axis of symmetry, and the maximum or minimum value of the function.

Completing the Square:

$(x-3)^2$ and $(x+7)^2$ are examples of **perfect squares**.

1. Expand the following perfect squares:

a. $(x-2)^2$

$= (x-2)(x-2)$

$= x^2 - 2x - 2x + 4$

$= x^2 - 4x + 4$

b. $(x-1)^2$

$= (x-1)(x-1)$

$= x^2 - x - x + 1$

$= x^2 - 2x + 1$

c. $(x+8)^2$

$= (x+8)(x+8)$

$= x^2 + 8x + 8x + 64$

$= x^2 + 16x + 64$

d. $(x+c)^2$

$= (x+c)(x+c)$

$= x^2 + cx + cx + c^2$

$= x^2 + 2cx + c^2$

2. Factor the following into perfect squares:

a. $x^2 - 6x + 9$

$= (x-3)(x-3)$

$= (x-3)^2$

b. $x^2 - 2x + 1$

$= (x-1)^2$

$\frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

c. $x^2 + 14x + 49$

$= (x+7)^2$

d. $x^2 + 3x + \frac{9}{4}$

$= \left(x + \frac{3}{2}\right)^2$

The process of adding a constant term to a quadratic expression to make it a perfect square is called **completing the square**.

3. Complete the square, then factor:

a. $x^2 + 6x + \underline{9} = (x+3)^2$

b. $x^2 - 5x + \underline{25/4} = (x - 5/2)^2$

$\left(\frac{-5}{2}\right)^2 = \frac{25}{4}$

c. $x^2 - 8x + \underline{16} = (x-4)^2$

$-8 \div 2 = -4$

$(-4)^2 = 16$

d. $x^2 + 11x + \underline{121/4} = \left(x + \frac{11}{2}\right)^2$

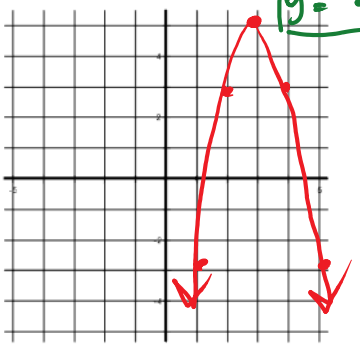
$\left(\frac{11}{2}\right)^2 = \frac{121}{4}$

To graph $y = ax^2 + bx + c$, **change to the form** $y = a(x-p)^2 + q$ **by completing the square.**

Example 1: Graph $y = -2x^2 + 12x - 13$ leave this for now

1. $-2(x^2 - 6x) - 13$
2. $-2(x^2 - 6x + 9 - 9) - 13$
3. $-2(x^2 - 6x + 9) + 18 - 13$
4. $-2(x^2 - 6x + 9) + 5 \Rightarrow \boxed{y = -2(x-3)^2 + 5}$

1. Remove the coefficient of x^2 as a common factor from the first two terms.
2. Complete the square. Add and subtract this number inside the brackets.
3. Remove the last term from the brackets and combine with the constant term.
4. Factor the trinomial square.



Example 2:

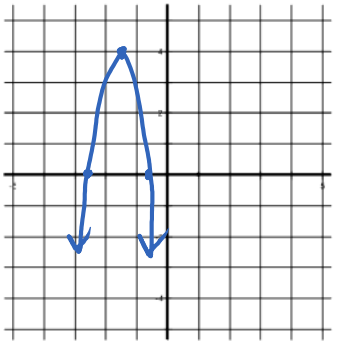
a. Graph $y = -4x^2 - 12x - 5$.

$y = -4(x^2 + 3x) - 5$

$y = -4\left(x^2 + 3x + \frac{9}{4} - \frac{9}{4}\right) - 5$

$y = -4\left(x^2 + 3x + \frac{9}{4}\right) + 9 - 5$

$y = -4\left(x + \frac{3}{2}\right)^2 + 4$



b. Determine the maximum or minimum value of y.

maximum when $y = 4$

c. For what value of x does the max or min occur?

This occurs when x is $-3/2$.

Example 3: Determine the equation of the axis of symmetry of the parabola with equation $y = -2x^2 + 5x - 3$.

$-\frac{5}{2} \div 2 = -\frac{5}{4}$ $\quad \left(-\frac{5}{4}\right)^2 = \frac{25}{16}$

$y = -2\left(x^2 - \frac{5}{2}x\right) - 3$

$= -2\left(x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16}\right) - 3$

$= -2\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) + \frac{25}{8} - 3$

$= -2\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) + \frac{1}{8}$

$y = -2\left(x - \frac{5}{4}\right)^2 + \frac{1}{8}$

$\boxed{x = \frac{5}{4}}$

Example 4: Determine the y-coordinate of the vertex of the graph of $y = -\frac{1}{2}x^2 - 3x - 8$.

$y = -\frac{1}{2}(x^2 + 6x) - 8$

$= -\frac{1}{2}(x^2 + 6x + 9 - 9) - 8$

$= -\frac{1}{2}(x^2 + 6x + 9) + \frac{9}{2} - 8$

$= -\frac{1}{2}(x + 3)^2 - \frac{7}{2}$

vertex $(-3, -7/2)$

y-value $= -7/2$

Assignment: pg. 192 #~~2, 3, 7, 8, 12~~

2(ac), 3(ac), 4(ac), 7(acd), 8, 12