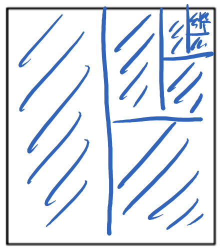
Lesson 1.5

Friday, February 3, 2017 5:42 PM

Zeno's Paradox: If you can only cover half distance will you ever get to end?

doesn't stop.
∴ infinite terms

Another example:



$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$$

Since this involves a series where there is always a next term, this is an example of an **infinite geometric series**.

eg: $(0.5)^{10} = 0.00097\ldots$

$(0.5)^{100} = 0.000\ 000\ 000\ 000\ 000\ 000000\ 000\ 000\ \ldots\ 78$

We know: $S_n = \dfrac{t_1(r^n - 1)}{r-1}$

$$S_n = \frac{t_1 r^n - t_1}{r-1}$$

$$= \frac{t_1 r^n}{r-1} - \frac{t_1}{r-1}$$

This becomes zero as n gets large

(If $-1 < r < 1$, then r^n becomes smaller and smaller as n increases. Eventually it becomes so small we can consider it equal to 0.)

→ a decimal/fraction

$$= 0 - \frac{t_1}{r-1}$$

$$\Rightarrow S_n = \frac{-t_1}{r-1} \text{ or } \frac{t_1}{1-r}$$

$$\boxed{\therefore S_n = \frac{t_1}{1-r}}$$

If $-1 < r < 1$
The series converges
(has a finite sum)

If $|r| > 1$
The series diverges
(keeps getting bigger)

Example 1: Determine whether each infinite geometric series converges or diverges. If it converges, determine its sum.

a. $27 - 9 + 3 - 1 + \ldots$

$r = \frac{-9}{27} = -\frac{1}{3}$

$-1 < -\frac{1}{3} < 1$

$\therefore$ converges

$S = \frac{t_1}{1-r} = \frac{27}{1-(-\frac{1}{3})} = \frac{27}{\frac{4}{3}} = 27 \times \frac{3}{4}$

$= \frac{81}{4}$

$= 20\frac{1}{4}$

b. $4 - 8 + 16 - 32 + \ldots$

$r = \frac{-8}{4} = -2$

not between -1 & 1

$\therefore$ diverges

Example 2: Determine a fraction that is equal to $0.4\overline{9}$.

$0.49999999\ldots = 0.4 + 0.09 + 0.009 + 0.0009 + \ldots$

$= 0.4 + \underbrace{0.09 + 0.09(0.1) + 0.09(0.1)^2 + \ldots}_{r = 0.1}$

$= 0.4 + \frac{0.09}{1-0.1}$ $\quad r < 1 \Rightarrow$ convergent

$= 0.4 + \frac{0.09}{0.9} = 0.4 + 0.1 = 0.5 = \frac{1}{2}$

Example 3: The first term of a geometric series is 2 and the sum to infinity is 4. Determine the common ratio.

$t_1 = 2$

$S = 4$

$S = \frac{t_1}{1-r}$

$(1-r) \times 4 = \frac{2}{\cancel{1-r}} \cancel{\times (1-r)}$

$4(1-r) = 2$

$4 - 4r = 2$

$-4r = 2 - 4$

$-4r = -2$

$r = \frac{1}{2}$

Assignment: Pg. 63 #~~1-8~~, 10, 14-16

#1-2 (a, d)
#3-4
#5 a, c

6-8, 10, 14, 15, 16

*Quiz 1.3-1.4 tomorrow