Lesson 1.4

Friday, February 3, 2017 5:42 PM

PREC 11 1.4 Geometric Series

Sequence: 2, 4, 8, 16, 32, … $r = 2$

Series: $2 + 4 + 8 + 16 + 32 \ldots$ $r = 2$

A **geometric series** indicates that the terms of a geometric sequence are to be added.

$S_1 = 2$ $S_2 = 6$ $S_3 = 14$ etc..

Example 1: Find the sum of the first 9 terms of the geometric series 2 + 6 + 18 + 54 + … .

Find t_9: $t_9 = 2 \times (3)^{9-1}$

$t_1 = 2$ $= 2 \times (3)^8$

$r = 3$ $= 13122$

$S_9 = 2 + 6 + 18 + 54 + \ldots + 13122$

Geometric = multiply ∴ multiply by r

$3S_9 = 6 + 18 + 54 + 162 + \ldots + 13122 + 39366$

$-\ S_9 = 2 + 6 + 18 + 54 \ldots + 13122$

$2S_9 = 39366 - 2$

$2S_9 = 39364$

$S_9 = 19682$

Example 2: Find the sum of the first 10 terms of the geometric series 2 + 4 + 8 + 16 + … .

$t_{10} = 2 \cdot 2^9 = 1024$

$r = 2$

$2\,S_{10} = 4 + 8 + 16 + 32 + \ldots + 2048$

$-\ S_{10} = 2 + 4 + 8 + 16 + \ldots + 1024$

$S_{10} = 2048 - 2$

$S_{10} = 2046$

In general to find the sum of $t_1 + t_1 r + t_1 r^2 + t_1 r^3 + \ldots + t_1 r^{n-1}$

$$rS_n = t_1 r + t_1 r^2 + t_1 r^3 + \cdots + t_1 r^{n-1} + t_1 r^n$$
$$-S_n = t_1 + t_1 r + t_1 r^2 + \cdots + t_1 r^{n-1}$$

$$rS_n - S_n = t_1 r^n - t_1$$
$$S_n(r-1) = t_1(r^n - 1)$$
$$\boxed{S_n = \frac{t_1(r^n - 1)}{r-1}}$$

Example 3: Find the sum of the first 15 terms of this geometric series: 40 – 20 + 10 – 5 + … to 2 decimal places.

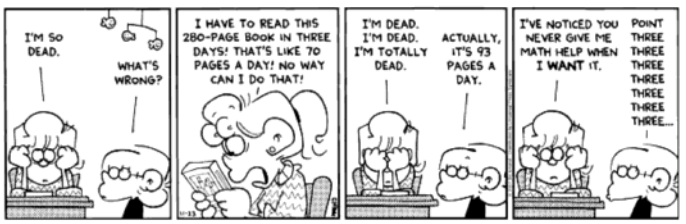
$t_1 = 40$
$n = 15$
$r = -\frac{1}{2}$ or -0.5

$$S_n = \frac{t_1(r^n - 1)}{r-1}$$
$$S_{15} = \frac{40[(-0.5)^{15} - 1]}{-0.5 - 1} = \frac{40(-1.000030518)}{-1.5}$$
$$= 26.67$$

Example 4: The sum of the first 14 terms of a geometric series is 16 383. The common ratio is -2. Determine the first term.

$S_{14} = 16383$
$r = -2$
$t_1 = ?$
$n = 14$

$$16383 = \frac{t_1[(-2)^{14} - 1]}{-2 - 1}$$
$$16383 = \frac{t_1[(-2)^{14} - 1]}{-3}$$
$$-49149 = t_1(16383)$$
$$\boxed{-3 = t_1}$$



$S_n = \frac{t_1(r^n - 1)}{r - 1}$

$t_1 = 6 \quad r = 2 \quad n = ?$

Example 5: Calculate the sum of 6 + 12 + 24 + 48 + … + 12 288.

$t_n = t_1 r^{n-1}$

$12\,288 = 6 \times 2^{n-1}$

$2048 = 2^{n-1}$

$2^{11} = 2^{n-1}$

$11 = n - 1$

$12 = n$

$S_{12} = \frac{6(2^{12} - 1)}{2 - 1}$

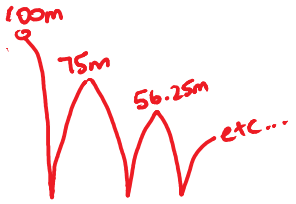
$= \frac{6(4096 - 1)}{1}$

$= 6(4095)$

$S_{12} = 24570$

Example 6: A golf ball is dropped from the top of a building 100 m above a paved road. In each bounce the ball reaches a vertical height that is $\frac{3}{4}$ the previous height.

a. Determine the vertical height (to the nearest tenth of a metre) of the ball after the seventh bounce (to the nearest tenth).



8th term

$t_8 = 100(3/4)^{8-1}$

$= 100(0.75)^7$

$= 13.348$

$= 13.3\,m$

b. Determine the total distance (to the nearest tenth) travelled by the ball when it contacts the floor for the seventh time.

$t_1 = 75$
$r = 0.75$
$n = 6$

after 100m, the ball goes up and down.
So double that part + add 100.

$\text{Distance} = 100 + 2\left(\frac{75(0.75^6 - 1)}{0.75 - 1}\right)$

$= 100 + 2\left(\frac{-61.651611\,33}{-0.25}\right)$

$= 593.2\,m$

Assignment: pg. 53 #1-4 (a, c), 5-8, 10, 11, 21

#1 visually.
#2-4 (a, c)
5-8, 10, 11, 18, 21