Lesson 1.3

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PREC 11 **1.3 Geometric Sequences**

Look at the following sequences:

1. 2, 10, 50, 250, … multiply by 5 $\therefore$ Not arithmetic
2. 3, -12, 48, -192, … multiply by -4 $\therefore$ Not arithmetic

Each successive term is found by multiplying by a constant. This constant is called the common ratio. A sequence where each term is obtained by multiplying the preceding term by a constant is called a geometric sequence.

Example 1: Find the common ratio *r*:

a) 12, 6, 3, 1.5, … $r = 1/2$ or 0.5

b) -2, 6, -18, 54, … $r = -3$

c) $a, ax, ax^2, ax^3, \ldots$ $r = x$

To find "r" divide a term by the previous one.

a) $\frac{6}{12} = \frac{1}{2}$

Example 2: For 6, 12, 24, …, determine:

$t_1 = 6$
$r = 2$

a. t_{10}

$t_1 = 6$

$t_2 = 12 = 6 \times 2$

$t_3 = 24 = 6 \times 2 \times 2$

$t_4 = 48 = 6 \times 2 \times 2 \times 2$
$= 6 \times 2^3$

$\vdots$

$t_{10} = 6 \times 2^9 = 3072$

b. t_n

$t_1 = 6$

$\vdots$

$t_n = 6 \times 2^{n-1}$

The general term of a geometric sequence is given by:

$$\boxed{t_n = t_1 \times r^{n-1}}$$

Example 3: Consider 3, 6, 12, 24, …

a. Determine t_{14}.

$t_1 = 3$
$t_{14} = ?$
$n = 14$
$r = 2$

$$t_n = t_1 \times r^{n-1}$$
$$t_{14} = 3 \times 2^{(14-1)}$$
$$= 3 \times 2^{13}$$
$$= 24576$$

b. Which term is 384?

$t_n = 384$
$n = ?$
$t_1 = 3$
$r = 2$

$$t_n = t_1 \times r^{n-1}$$
$$\frac{384}{3} = \frac{\cancel{3} \times 2^{n-1}}{\cancel{3}}$$
$$128 = 2^{n-1}$$

Note
$(2^7 = 128)$

$$2^7 = 2^{n-1}$$
$$7 = n - 1$$
$$8 = n$$
$$t_8 = 384$$

Example 4: The terms placed between two non-consecutive terms of a geometric sequence are called **geometric means**. Insert 4 geometric means between 81 and $\frac{1}{729}$.

$$81 \xrightarrow{\times r} \underline{9} \xrightarrow{\times r} \underline{1} \xrightarrow{\times r} \underline{\tfrac{1}{9}} \xrightarrow{\times r} \underline{\tfrac{1}{81}} \xrightarrow{\times r} \frac{1}{729}$$

$$\frac{\cancel{81}\, r^5}{\cancel{81}} = \frac{1}{729} \times \frac{1}{81}$$
$$\sqrt[5]{r^5} = \sqrt[5]{\frac{1}{59049}}$$
$$r = \frac{\sqrt[5]{1}}{\sqrt[5]{59049}}$$
$$r = \frac{1}{9}$$

calc: $\boxed{59049^{\wedge}(1/5)}$

Example 5: In a geometric sequence $t_1 = 5$ and $t_5 = 1280$.

a. Determine t_2 and t_6.

$t_5 = t_1 \cdot r^{5-1}$

$1280 = 5 \cdot r^4$

$256 = r^4$

$4 = r$

$t_2 = t_1 \cdot r$

$= 5 \cdot (4)$

$= 20$

$t_6 = t_1 \cdot r^{6-1}$

$= 5 \cdot (4)^5$

$= 5120$

b. The last term of the sequence is 20480. How many terms are in this sequence?

$t_1 = 5$

$t_n = 20480$

$n = ?$

$r = 4$

$t_n = t_1 \cdot r^{n-1}$

$20480 = 5(4)^{n-1}$

$4096 = 4^{n-1}$

$4^6 = 4^{n-1}$

$6 = n - 1$

$7 = n$

There are 7 terms.

$t_7 = 20480$

Example 6: Three consecutive terms of a geometric sequence are $x+3$, x, and $x-5$. Determine the value of x and the three terms.

$\times r \quad \times r$

$$\cancel{(x+3)}\frac{x}{\cancel{x+3}} = \frac{x-5}{x}(x+3)$$

$$x \cdot x = \frac{(x-5)(x+3)}{\cancel{x}}\cancel{x}$$

$$x^2 = (x-5)(x+3)$$

$$\cancel{x^2} = \cancel{x^2} + 3x - 5x - 15$$

$$-\cancel{x^2} \quad -\cancel{x^2}$$

$$0 = -2x - 15$$

$$2x = -15$$

$$x = -15/2 \text{ or } -7.5$$

$x+3,\ x,\ x-5$

$\therefore -7.5+3, -7.5, -7.5-5$

$\boxed{\therefore -4.5, -7.5, -12.5}$

Assignment: pg. 39 #1-6, 9, 10, ~~12~~, ~~18~~, 23, 19