Lesson 1.2

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PREC 11 **1.2 Arithmetic Series**

A **series** is a sum of the terms in a sequence.

Eg. 5, 8, 11, 14, … $d=3$ $t_1=5$, $t_2=8$, $t_3=11$ …

5+8+11+14+… $d=3$ $S_1=5$, $S_2=13$, $S_3=24$ …

The term S_n is used to represent the sum of the first n terms of a series.

Example 1: Determine the sum of the first 14 terms of the arithmetic series 9 + 15 + 21 + …

What is t_{14}? $t_{14} = 9+(13)(6)$ $d=6$

$= 87$

$S_{14} = 9+15+21+\dots+87$

$+ S_{14} = 87+81+\dots+9$

$2S_{14} = 96+96+\dots+96$

$2S_{14} = 14(96)$

$S_{14} = 7(96)$

$S_{14} = 672$

∴ Sum of first 14 terms is 672.

In general, $S_n = t_1 + (t_1+d) + (t_1+2d) + \dots + (t_n-d) + t_n$

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$+ S_n = t_n + (t_n-d) + (t_n-2d) + \dots + (t_1+d) + t_1$

$2S_n = (t_1+t_n) + (t_1+t_n) + (t_1+t_n) + \dots + (t_1+t_n) + (t_1+t_n)$

$2S_n = n(t_1+t_n)$

$S_n = \frac{n}{2}(t_1+t_n)$

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Also, if $S_n = \frac{n}{2}(t_1 + t_n)$ and $t_n = t_1 + (n-1)d$ then:

substitute

$S_n = \frac{n}{2}(t_1 + t_1 + (n-1)d)$

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$

Therefore, the sum of n terms of an arithmetic series is given by:

$S_n = \frac{n}{2}(t_1 + t_n)$ Use if you know both t_1 + t_n

OR

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$ Use if you know both t_1 and d.

Example 2: Determine the sum of the first 6 terms of this arithmetic series:

$-75 - 69 - 63 - 57 - 51 - 45 - \ldots$

$S_6 = \frac{6}{2}(-75 + -45)$

$= 3(-120)$

$= -360$

Example 3: An arithmetic series has $t_1 = 5.5$ and $d = -2.5$; determine S_{40}.

$S_{40} = \frac{n}{2}(2t_1 + (n-1)d)$

$= \frac{40}{2}(2(5.5) + (40-1)(-2.5))$

$= 20(11 + (39)(-2.5))$

$= 20(11 - 97.5)$

$= 20(-86.5)$

$= -1730$

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Example 4: An arithmetic series has $S_{20} = 143\frac{1}{3}$, $d = \frac{1}{3}$, and $t_{20} = 10\frac{1}{3}$. Determine the first 3 terms of the series.

$S_n = \frac{n}{2}(t_1 + t_n) \Rightarrow S_{20} = \frac{20}{2}(t_1 + t_{20})$

$143\frac{1}{3} = \frac{20}{2}(t_1 + 10\frac{1}{3})$

$143\frac{1}{3} = 10(t_1 + 10\frac{1}{3})$

$14\frac{1}{3} = t_1 + 10\frac{1}{3}$

$4 = t_1$

$\therefore 4 + 4\frac{1}{3} + 4\frac{2}{3}$

Example 5: The sum of the first two terms of an arithmetic series is 19 and the sum of the first four terms is 50. $S_2 = 19$ $S_4 = 50$

a. What are the first six terms of the series?

$S_2 = \frac{2}{2}(2t_1 + (2-1)d)$

$19 = 1(2t_1 + d)$

$19 = 2t_1 + d$

$S_4 = \frac{4}{2}(2t_1 + (4-1)d)$

$50 = 2(2t_1 + 3d)$

$50 = 4t_1 + 6d$

elimination

$2\times(19 = 2t_1 + d)$

$50 = 4t_1 + 6d$

$38 = 4t_1 + 2d$

$-(50 = 4t_1 + 6d)$

$-12 = -4d$

$3 = d$

Substitution

$19 - 2t_1 = d$

$50 = 4t_1 + 6(19 - 2t_1)$

$50 = 4t_1 + 114 - 12t_1$

$-64 = -8t_1$

$8 = t_1$

$\therefore 8 + 11 + 14 + 17 + 20 + 23$

b. What is the sum of the first 20 terms?

$S_{20} = \frac{n}{2}(2t_1 + (n-1)d)$

$= \frac{20}{2}(2(8) + (20-1)(3))$

$= 10(16 + 19(3))$

$= 10(16 + 57)$

$= 10(73)$

$= 730$

Example 6: Students created a trapezoid from cans. The bottom row had 49 cans, and each consecutive row had 4 fewer cans than the previous row. There were 11 rows in the trapezoid. How many cans were there?

S_{11}?

$n=11$
$t_1=49$
$d=-4$

$49+45+41+\ldots$

$S_n = \frac{n}{2}(2t_1+(n-1)d)$

$S_{11} = \frac{11}{2}(2(49)+(11-1)(-4))$

$= 5.5(98+(10)(-4))$
$= 5.5(98-40)$
$= 5.5(58)$
$= 319$

Assignment: ~~pg. 27 #1-6 (a, c), 7, 9, 11, 15, 20~~

HW
p. 27 #1-6 (a, c), 7, 9, 11, 15, 20, 21