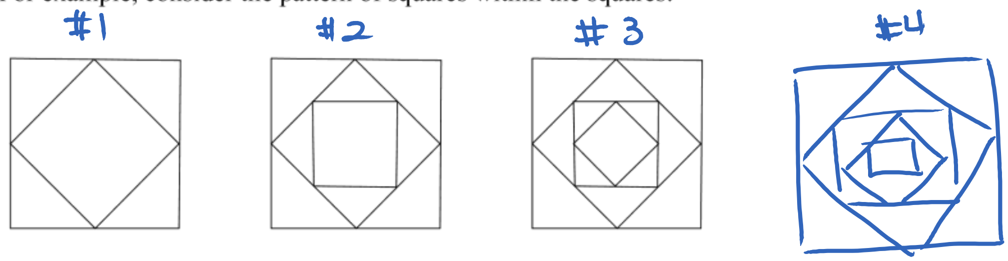
Lesson 1.1

Friday, February 3, 2017 5:41 PM

1.1 Arithmetic Sequences

In this unit we investigate patterns to explore types of sequences.

For example, consider the pattern of squares within the squares:



Draw the next figure, and fill in the chart below:

Diagram number	1	2	3	4
Number of Triangles	4	8	12	16

The next term for the number of triangles can be found by adding 4 .

This is an example of an arithmetic sequence.
(adding / subtracting)

Other examples of arithmetic sequences are:

- 4, 7, 10, 13, 16, … add 3
- 5, -1, -7, -13, -19, … subtract 6 / add -6
- 3, 5, 7, 9, 11, … add 2

What pattern do you see that would help you recognize whether or not a sequence is an **arithmetic** sequence?

you must add (or subtract) the same value to each term to get the next term.

An **arithmetic sequence** is an ordered list of elements where the difference between consecutive terms is constant. This constant value is called the common difference ("d")

- The first term of the sequence is t_1 or "a".
- The number of terms in the sequence is n.
- The **general term** of the sequence is t_n.

Consider the sequence: 5, 8, 11, 14, …

arithmetic sequence with common difference "d" of 3

$t_1 = 5$

$t_2 = 8 = 5+3$

$t_3 = 11 = 5+3+3 = 5+2(3)$

$t_4 = 14 = 5+3+3+3 = 5+3(3)$

⋮

$t_{20} = 5+19(3) = 62$

∴ For an arithmetic sequence:

$$t_n = t_1 + (n-1)d$$

Example 1: For the arithmetic sequence: -3, 2, 7, 12, …

a. Determine t_{20}.

$t_1 = -3$

$n = 20$

$d = 5$

$t_n = t_1 + (n-1)d$

$t_{20} = -3 + (20-1)(5)$

$= -3 + (19)(5)$

$= -3 + 95 = 92$

b. Which term in the sequence has the value 212?

$t_n = 212$

$t_1 = -3$

$n = ?$

$d = 5$

$t_n = t_1 + (n-1)d$

$212 = -3 + (n-1)5$

$215 = (n-1)5$

$43 = n-1$

$44 = n$

$\boxed{\therefore t_{44} = 212}$

Example 2: Find the number of terms in the arithmetic sequence 3, -1, -5, …, -117.

$t_1 = 3$
$t_n = -117$
$n = ?$
$d = -4$

$-117 = 3 + (n-1)(-4)$
$-120 = (n-1)(-4)$
$30 = n-1$
$31 = n$

$\boxed{\therefore 31 \text{ terms}}$

Example 3: Two terms in an arithmetic sequence are $t_3 = 4$ and $t_8 = 34$. What is t_1?

$t_n = t_1 + (n-1)d$

$$\begin{cases} t_3 = t_1 + (3-1)d \\ t_8 = t_1 + (8-1)d \end{cases}$$

$$\begin{cases} 4 = t_1 + 2d \\ 34 = t_1 + 7d \end{cases}$$

Elimination
$4 = t_1 + 2d$
$-(34 = t_1 + 7d)$
$-30 = -5d$
$6 = d$

or Substitution
$t_1 = 4 - 2d$
$34 = (4-2d) + 7d$
$34 = 4 - 2d + 7d$
$30 = 5d$
$6 = d$

$4 = t_1 + 2(6)$
$-8 = t_1$

Example 4: Some comets are called periodic comets because they appear regularly in our solar system. The comet Kojima appears about every seven years and was last seen in 2007. Halley's Comet appears about every 76 years and was last seen in 1986. Determine whether or not both comets should appear in 3034.

Kojima: $3034 = 2007 + (n-1)7$
$1027 = (n-1)7$
$146.7 = n-1$
$147.7 = n$

Halley's: $3034 = 1986 + (n-1)(76)$
$1048 = (n-1)(76)$
$13.789 = n-1$
$14.789 = n$

← Decimal →
↳ should not appear.

Example 5: The terms placed between two non-consecutive terms of an arithmetic sequence are called **arithmetic means**. Place three arithmetic means between -4 and 8.

$-4 \quad \underline{-1} \quad \underline{2} \quad \underline{5} \quad 8$

$+3 \quad +3 \quad +3 \quad +3$

$t_1 = -4$

$t_5 = 8$

$n = 5$

$d = ?$

$t_5 = t_1 + (n-1)d$

$8 = -4 + (5-1)d$

$8 = -4 + 4d$

$12 = 4d$

$3 = d$

Example 6: Consider the sequence $x+2, 3x-1, 2x+1$.

$+d \quad +d$

a. Determine the value of x such that this forms an arithmetic sequence.

$(3x-1) - (x+2) = (2x+1) - (3x-1)$

$2x - 3 = -x + 2$

$3x - 3 = 2$

$3x = 5$

$x = \frac{5}{3}$

b. Determine the numerical values of the 3 terms.

$x+2$	, $3x-1$	, $2x+1$
$\frac{5}{3} + 2$	$3\left(\frac{5}{3}\right) - 1$	$2\left(\frac{5}{3}\right) + 1$
$\frac{5}{3} + \frac{6}{3}$	$\frac{15}{3} - \frac{3}{3}$	$\frac{10}{3} + 1$
$\frac{11}{3}$	$\frac{12}{3}$	$\frac{13}{3}$

Assignment: pg. 16 #1-12, 21, 26

$= \frac{11}{3}, 4, \frac{13}{3}$