9.3 Quadratic inequalities in Two Variables

Warm Up 1: Are the following points solutions to the inequality $\geq x^2 - 3x - 4$.

a) (3, 0)

LS $0 \geq 3^2 - 3(3) - 4$ RS

$0 \geq 9 - 9 - 4$

$0 \geq -4$ ✓

yes.

b) (7, -1)

$-1 \geq 7^2 - 3(7) - 4$

$-1 \geq 24$ ✗

NO.

Warm up 2: For A(-1, a) to be a solution of $y > -2x^2 + 5$, what must be true about a? How could you write this in set notation?

point

$a > -2(-1)^2 + 5$

$a > 3$

a has to be greater than 3.

$\{a \mid a > 3, a \in \mathbb{R}\}$

A quadratic inequality in two variables can be written as:

$y > ax^2 + bx + c$ $y < ax^2 + bx + c$ $y \geq ax^2 + bx + c$ $y \leq ax^2 + bx + c$

leading coefficient

Where a, b and c are constants and $a \neq 0$. The graph of a quadratic inequality is all the ordered pairs (x, y) that satisfy the inequality.

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Example 1: Graph $y < x^2 - 6x$ and identify two solutions points.

Boundary Line

$y = x^2 - 6x$

x-int:

$0 = x^2 - 6x$

$0 = x(x-6)$

$x = 0$ or $x = 6$

vertex:

$y = 3^2 - 6(3)$

$y = -9$

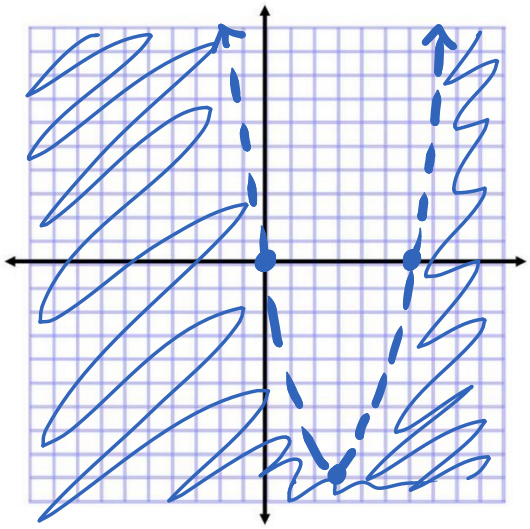
$(3, -9)$

Broken line b/c $<$

Test pt. $(5,5)$

$5 < 5^2 - 6(5)$

$5 < -5$ ×



Two soln: $(-5,5)$ & $(8,8)$

Example 2: Graph $y \leq -2x^2 + 8$

B.L. $y = -2x^2 + 8$

x-int:

$0 = -2x^2 + 8$

$-8 = -2x^2$

$4 = x^2$

$\pm 2 = x$

y-int / vertex:

$y = -2(0)^2 + 8$

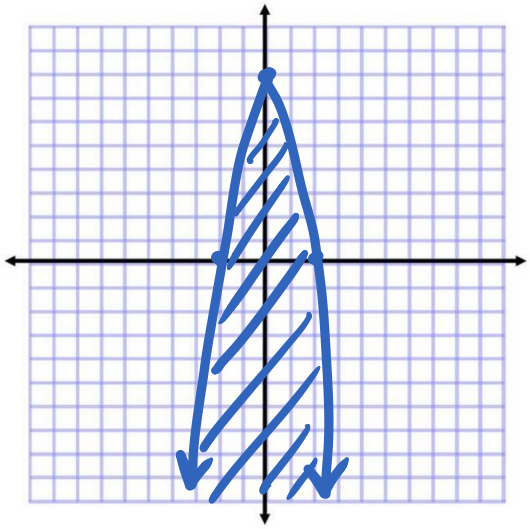
$y = 8$

Solid b/c $\leq$

Test pt. $(0,0)$

$0 \leq -2(0)^2 + 8$

$0 \leq 8$ ✓



Example 3: Two numbers are related in this way: four times the square of one number is less than 2 times the sum of the other number and 3.

a) Write an inequality to represent this situation.

Let x = one #

y = other #

$4x^2 < 2(y+3)$

$4x^2 < 2y+6$

b) Graph the inequality.

B.L. $4x^2 = 2y+6$

$4x^2 - 6 = 2y$

$2x^2 - 3 = y$

x-int: $2x^2 - 3 = 0$

$2x^2 = 3$

$x = \pm\sqrt{1.5}$

y-int/vertex

$2(0)^2 - 3 = y$

$-3 = y$

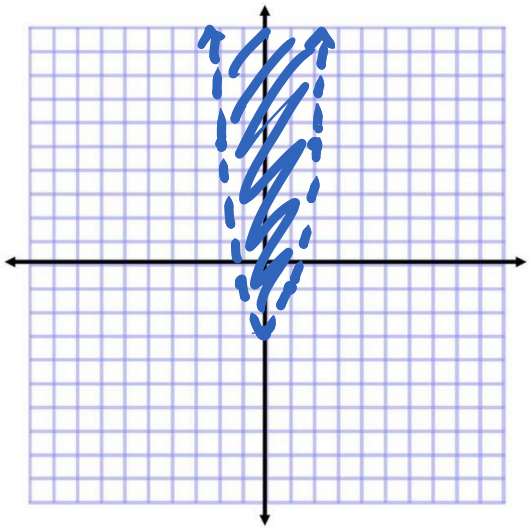
$(0, -3)$

Broken line (<)

Test pt $(0,0)$

$4(0)^2 < 2(0)+6$

$0 < 6$ ✓



c) Use the graph to identify three pairs of integer values for the two numbers.

$x = 1$ and $y = 2$

$x = 0$ and $y = 0$

$x = 0$ and $y = 2$

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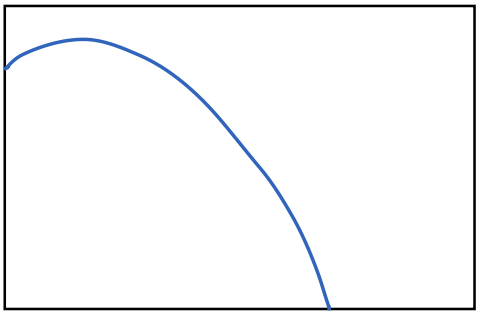
Example 4:

In order to get the most revenue from registrations for a camping trip, an adventure company needs to have as many campers as possible at a price per camper that is reasonable. If 15 people sign up, the price per person is $50. The registration fee is reduced by $2 for each additional camper beyond 15. The relationship between the number of campers beyond 15 and y is the given by $y \leq (50 - 2x)(15 + x)$, where x represents the number of campers beyond 15 and y is the total revenue, in dollars.

x = # of $2 decrease

a) Use your calculator to sketch a graph to represent this situation.

$y_1 = (50-2x)(15+x)$



Xmin: 0
X max: 50
y min: 0
ymax: 1000

What do we need to change in our calculator to get it to shade the solution region?

Move the cursor to the left of y
press enter button 2x for ≥
press enter button 3x for ≤

b) Use your calculator to determine the total number of registrations that will generate revenue of at least $500.

$y_2 = 500$

(17.25, 500)

$15 + 17.25 = 32.25$

≈ 32

32 registrations will generate revenue of at least $500.