9.2 Quadratic Inequalities in One Variable

A quadratic inequalities in one variable can be written as:

$ax^2 + bx + c < 0$ $\qquad ax^2 + bx + c > 0$ $\qquad ax^2 + bx + c \leq 0$ $\qquad ax^2 + bx + c \geq 0$

Where a, b and c are constants and $a \neq 0$.

When solving a quadratic inequality in one variable, there are more than one method you can use: graphing or test point chart.

method 1 method 2

Example 1: Solve $x^2 - x - 12 \leq 0$

Step 1: Rewrite the inequality as a quadratic equation and find its roots.

$x^2 - x - 12 = 0$

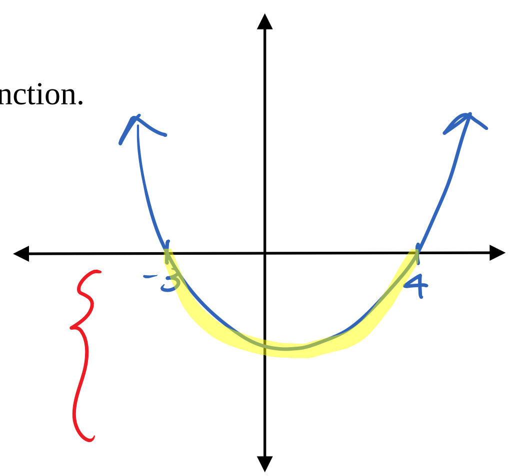
$(x-4)(x+3) = 0$

$x = 4, -3$

** These are critical points to solve your inequality **

Step 2: Use the roots to sketch a graph of the quadratic function.

leading coefficient "a" >0



below x-axis ≤ 0

$\underbrace{x^2 - x - 12}$ ≤ 0

Step 3: Use the critical points and your inequality sign to shade the appropriate region on a number line.

$\leq$ closed circle

Step 4: Write the solution to the inequality in set notation.

$\{x \mid -3 \leq x \leq 4,\ x \in \mathbb{R}\}$

Example 2: Solve $2x^2 - 5x - 3 > 0$

Step 1: Rewrite the inequality as a quadratic equation and find its roots.

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$2x+1=0$ $\quad$ $\boxed{x = 3}$

$\boxed{x = -\frac{1}{2}}$

Step 2: Use the roots to complete the following chart.

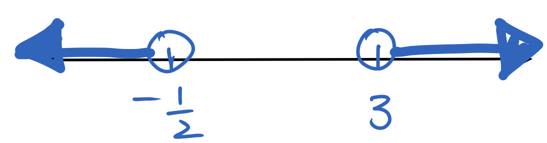
Interval	$(-\infty, -\frac{1}{2})$ $x < -\frac{1}{2}$	$(-\frac{1}{2}, 3)$ $-\frac{1}{2} < x < 3$	$(3, \infty)$ $3 < x$ or $x > 3$
Test points	$x = -1$	$x = 2$	$x = 9$
Substitution	$2(-1)^2 - 5(-1) - 3$ 4	$2(2)^2 - 5(2) - 3$ -5	$2(9)^2 - 5(9) - 3$ 114
Is $2x^2 - 5x - 3 > 0$	yes	NO	yes

Step 3: Use the critical points and your inequality sign to shade the appropriate region on a number line.

$2x^2 - 5x - 3 > 0$

↑

open circle



Step 4: Write the solution to the inequality in set notation.

$\{x \mid x < -\frac{1}{2},\ x > 3,\ x \in \mathbb{R}\}$

Example 1 used graphing to find the solution to the inequality and Example 2 used a test point chart to organize the results. Which method do you prefer? Why?

Example 3: Solve $-8x \leq -3(x^2 - 1)$ using a method of your choice.

$-8x \leq -3x^2 + 3$

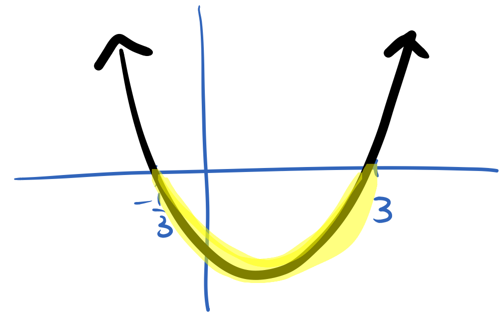
Using method 1

$3x^2 - 8x - 3 = 0$

$(3x+1)(x-3) = 0$

$3x + 1 = 0 \qquad x - 3 = 0$

$x = -\frac{1}{3} \qquad x = 3$



Leading coefficient "a" > 0
open up.

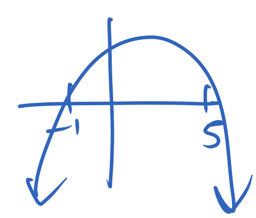
$0 \leq$

$0 \leq$

closed circle $\leq$

$\{x \mid -\frac{1}{3} \leq x \leq 3, x \in \mathbb{R}\}$

Example 4: Erik submitted the following number line as a solution to his quadratic inequality. What inequality was he solving?

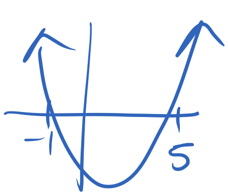


$x = -1 \qquad x = 5$

$x + 1 = 0 \qquad x - 5 = 0$

$(x+1)(x-5)$

Soln #2
$-x^2 + 4x + 5 \geq 0$



Soln #1 $x^2 - 4x - 5 \leq 0$

Example 5:

a) Solve $x^2 - 2x + 3 \leq 0$

Find the roots

$x^2 - 2x + 3 = 0$

Quad Formula

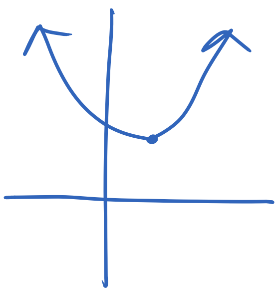
$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 12}}{2} \leftarrow \text{No real roots}$$

Complete square?

$$y = x^2 - 2x + 3$$
$$= (x^2 - 2x + 1 - 1) + 3$$
$$= (x^2 - 2x + 1) + 2$$
$$= (x-1)^2 + 2$$

vertex $(1, 2)$



The inequality does not work b/c $x^2 - 2x + 3$ is Never less than zero.

$\boxed{\therefore \text{ No sol}^{\underline{n}}}$

b) Solve $x^2 - 2x + 3 \geq 0$

always true

$\{x \mid x \in \mathbb{R}\}$

c) Compare your solutions for a and b. What does this mean?

When we cannot algebraically determine the roots, it does not necessarily mean there is no sol$^{\underline{n}}$.

Always use the graph to verify.

$A = lw$

Example 6: The length of a rectangle is 2 cm greater than its width. The area of the rectangle is at least 20 cm^2.

$l = w + 2$

a) Identify the variables and write an inequality to represent this situation.

let w = width

$w(w+2) \geq 20$

$w > 0$

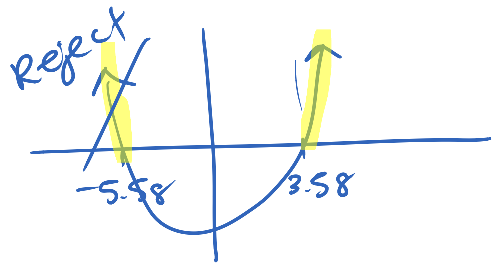
($w \neq$ negative #)

b) Use an algebraic method to determine the possible dimensions of the rectangle? Round your answer(s) to two decimal places.

$w(w+2) \geq 20$

$w^2 + 2w \geq 20$

$w^2 + 2w - 20 \geq 0$



$w^2 + 2w - 20 = 0$

$$w = \frac{-2 \pm \sqrt{2^2 - 4(1)(-20)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 + 80}}{2}$$

$$= \frac{-2 \pm \sqrt{84}}{2}$$

$$\left\{ \begin{aligned} l &= w + 2 \\ l &= 3.58 + 2 \\ &= 5.58 \end{aligned} \right.$$

$$\frac{-2 + \sqrt{84}}{2} = 3.58$$

$$\frac{-2 - \sqrt{84}}{2} = \cancel{-5.58}$$

Reject

$$\boxed{\begin{aligned} w &\geq 3.58\text{cm} \\ l &\geq 5.58\text{cm} \end{aligned}}$$

$\{w \mid w \geq 3.58, w \in \mathbb{R}\}$

$\{l \mid l \geq 5.58, l \in \mathbb{R}\}$

c) Could you use your calculator to verify your results? Explain.

yes

To visualize graph and find zero.