

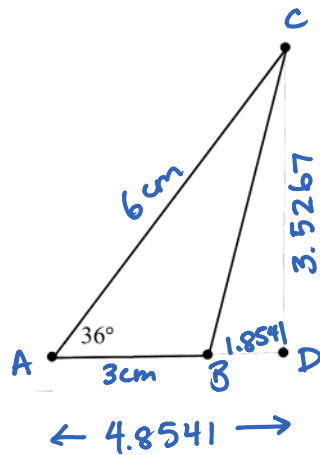
Lesson 2.4

Saturday, February 4, 2017 4:37 PM

PREC 11

2.4 The Cosine Law

Use SOH CAH TOA to determine the length of BC to the nearest tenth of a centimetre.



$$\sin 36^\circ = \frac{CD}{6}$$

$$6 \sin 36^\circ = CD$$

$$3.5267 = CD$$

$$\cos 36^\circ = \frac{AD}{6}$$

$$6 \cos 36^\circ = AD$$

$$4.8541 = AD$$

$$a^2 + b^2 = c^2$$

$$1.8541^2 + 3.5267^2 = BC^2$$

$$15.8753 = BC^2$$

$$3.9844 = BC$$

$$4.0 \text{ cm} = BC$$

$$4.8541 - 3 = 1.8541$$

This method is time-consuming. We can determine the length of BC in one step using the **Cosine Law**.

Cosine Law: In every triangle ABC,

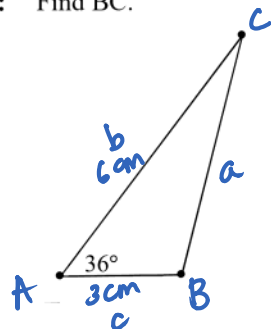
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Cosine Law \Rightarrow 2 sides + angle between
 \Rightarrow 3 sides given (no angle)

Example 1: Find BC.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 6^2 + 3^2 - 2(6)(3) \cos 36^\circ$$

$$a^2 = 36 + 9 - 36 \cos 36^\circ$$

$$a^2 = 45 - 36 \cos 36^\circ$$

$$a^2 = 15.8753...$$

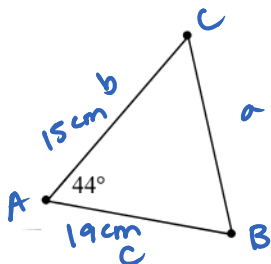
$$a = \sqrt{15.8753...}$$

$$a = 3.98439$$

$$a = 4.0 \text{ cm}$$

Example 2: In each triangle, determine the length of BC to the nearest tenth of a centimetre.

a.



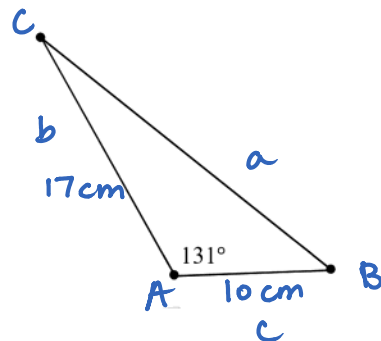
$$a^2 = 15^2 + 19^2 - 2(15)(19) \cos 44^\circ$$

$$a^2 = 175.9763...$$

$$a = \sqrt{175.9763...}$$

$$a = 13.3 \text{ cm}$$

b.



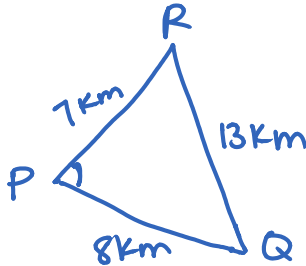
$$a^2 = 17^2 + 10^2 - 2(17)(10) \cos 131^\circ$$

$$a^2 = 612.0600...$$

$$a = \sqrt{612.06...}$$

$$a = 24.7 \text{ cm}$$

Example 3: Two ships set sail from port, P, heading in different directions. The first ship sails 7 km to R and the second ship sails 8 km to Q. If the distance between R and Q is 13 km, determine the angle between the directions of the two ships.



$$\begin{aligned}
 p^2 &= r^2 + q^2 - 2rq \cos P \\
 13^2 &= 8^2 + 7^2 - 2(8)(7) \cos P \\
 169 &= 64 + 49 - 112 \cos P \\
 169 &= 113 - 112 \cos P \\
 -113 &\quad -113
 \end{aligned}$$

$$\frac{56}{-112} = \frac{-112 \cos P}{-112}$$

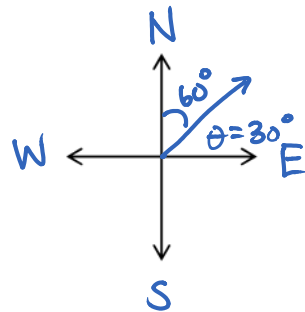
$$-0.5 = \cos P$$

$$\cos^{-1}(-0.5) = P = 120^\circ$$

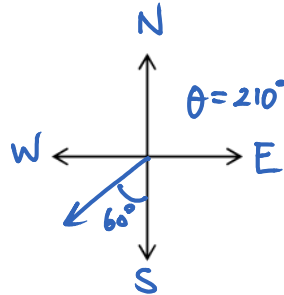
The angle between two ships is 120° .

Example 4: A retaining wall is leaning at an angle of 70° to the horizontal. A rigid support is to be placed 5.0 m from the base of the wall and it will be attached to the wall 2.5 m from the wall's base. Determine the length of the support to the nearest tenth of a metre and the measure of the angle between the support and the wall to the nearest degree.

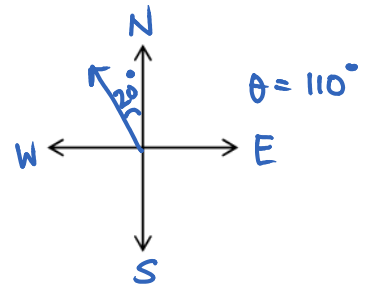
Bearings: A **bearing** of one point from a second point is a way of giving directions. Bearings are usually measured from North in a clockwise direction.



N 60° E



S 60° W

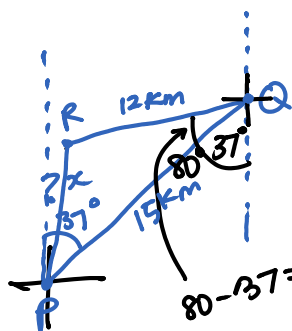


N 20° W

Example 5: A liner is leaving a port P and sails 15 km on a course of 37° to position Q. It then changes course to 270° and sails for 12 km to position R. Calculate the distance the liner must sail to return from R to P.

N 37° E

S 80° W



$$x^2 = 12^2 + 15^2 - 2(12)(15)\cos 43^\circ$$

$$x^2 = 105.7$$

$$x = 10.3 \text{ km}$$

Parallel lines
alt. int. \angle s



Example 6: From the top of a 30 m observation tower, a fire ranger observes smoke at a bearing of 90° with an angle of depression of 5° . The ranger spots more smoke at a bearing of 200° with an angle of depression of 2° . How far apart are the sources of smoke (to the nearest metre)?

Assignment: pg. 119 #1, 2 (ac), 3, 4ad, 6, 10, 14, 19, 20, 23, 25

