Lesson 2.3.2
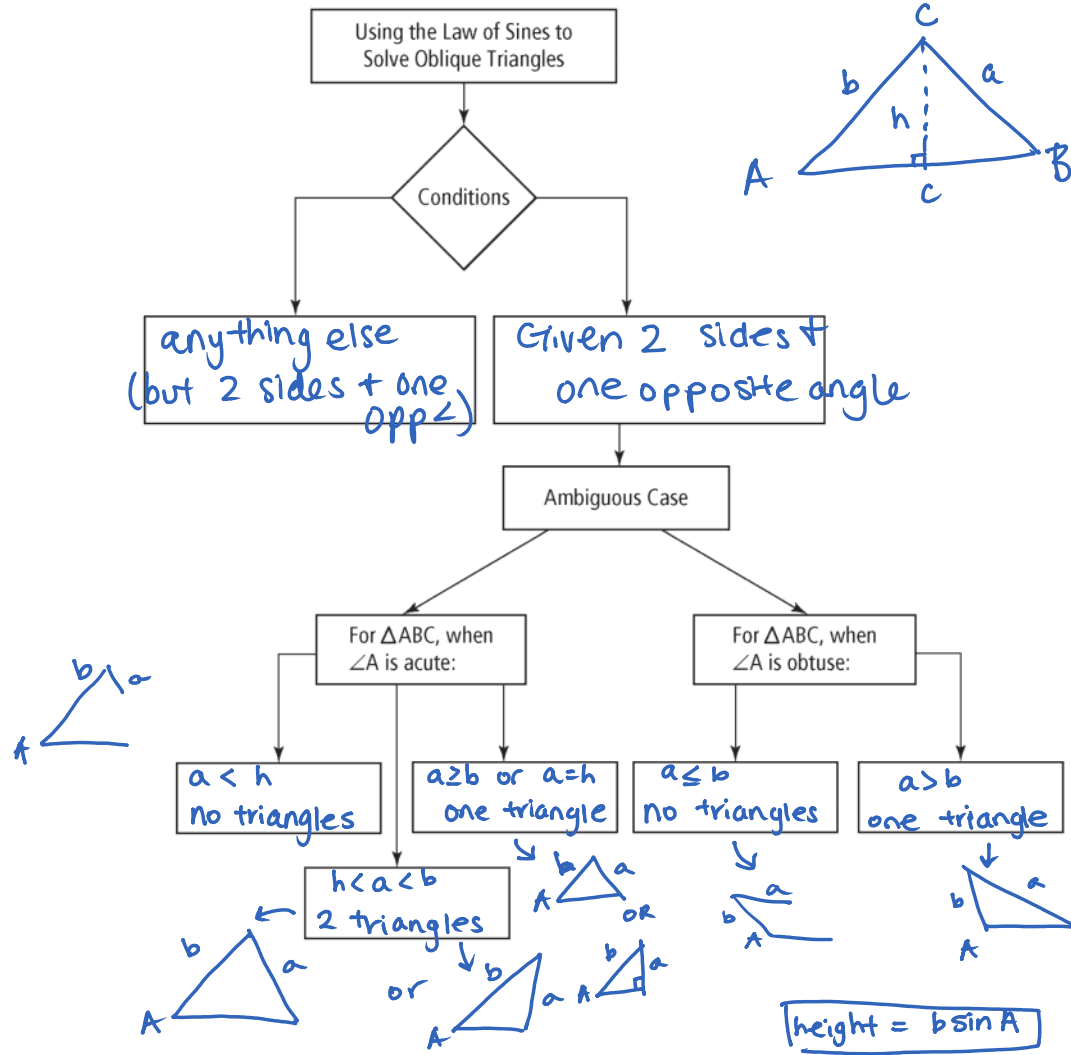
PREC 11   **2.3 The Sine Law (Ambiguous Case)**

Try This Question:

The Canadian Coast Guard Pacific Region is responsible for more than 27 000 km of coastline. The rotating spotlight from the Coast Guard ship can illuminate up to a distance of 250 m. An observer on the shore is 500 m from the ship. His line of sight to the ship makes a 20° with the shoreline. What length of shoreline is illuminated by the spotlight
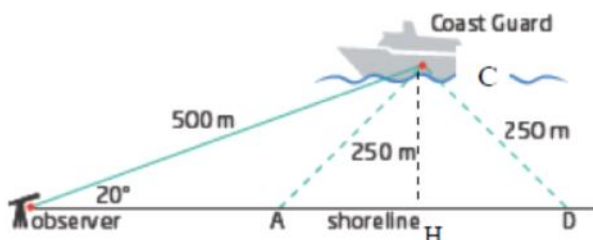
If you are given the measures of two sides and an angle opposite one of those sides, the **ambiguous case** may occur. In this instance, there might be one triangle, two triangles, or no triangle.

Using the Law of Sines to
Solve Oblique Triangles

Conditions

anything else
(but 2 sides + one
opp∠)

Given 2 sides +
one opposite angle

Ambiguous Case

For △ABC, when
∠A is acute:

For △ABC, when
∠A is obtuse:

a < h
no triangles

a≥b or a=h
one triangle

a≤b
no triangles

a>b
one triangle

h<a<b
2 triangles
or

height = b sin A

Example 1: For which of these triangles must you consider the ambiguous case?

  a.   In $\triangle ABC$, $a = 16, b = 12, c = 5$.

  3 sides → NO

  b.   In $\triangle DEF$, $\angle D = 112°, e = 110, f = 65$.

  angle not opposite side → NO

  c.   In $\triangle ABC$, $\angle B = 35°, a = 27, b = 21$.

  2 sides + 1 opposite angle → yes

  d.   In $\triangle DEF$, $\angle D = 108°, \angle E = 52°, f = 15$.

  2 angles + 1 side → NO

$\sin A = \dfrac{h}{10}$

↳ (2 sides + 1 opposite angle)

Example 2: Given each SSA situation for $\triangle ABC$, determine how many triangles are possible.

  a. $\angle A = 30°, a = 5, b = 10$.   b. $\angle A = 30°, a = 7, b = 10$.

$h = b \sin A$

$h = 10 \sin(30°)$

$h = 5$

$a = h$

∴ 1 triangle

$h = 5$

$b > a > h$

∴ Two triangles

  c. $\angle A = 30°, a = 12, b = 10$.   d. $\angle A = 30°, a = 4, b = 10$.

$h = 5$

$a \geq b$

∴ 1 triangle

$h = 5$

$a < h$

∴ No triangle

Example 3: In an extreme adventure triathalon, participants swim 1.7 km from a dock to one end of an island, run 1.5 km due north along the length of the island, and then kayak back to the dock. From the dock, the angle between the lines of sight to the ends of the island measures 15°. How long is the kayak leg of the race?
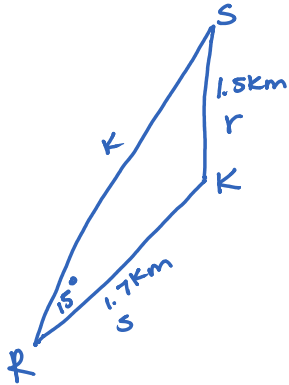
Check for ambiguous case

$$h = 1.7 \sin 15°$$
$$h = 0.44$$
$$1.7 > 1.5 > 0.44$$
$$b > a > h$$
$$\therefore 2 \text{ triangles}$$

Find ∠S:

$$\frac{\sin 15°}{1.5} = \frac{\sin S}{1.7}$$

$$\angle S = 17°$$
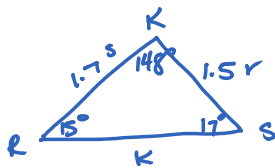
$$\angle K = 180° - 15° - 17°$$
$$= 148°$$

Find K:

$$\frac{\sin 15°}{1.5} = \frac{\sin 148°}{K}$$

$$\boxed{K = 3.1 km}$$
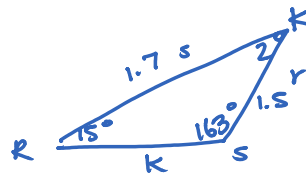
Find ∠S:

$$\angle S = 180 - 17$$
$$= 163°$$

$$\angle K = 180 - 15 - 163$$
$$= 2°$$

Find K:
$$\frac{\sin 15°}{1.5} = \frac{\sin 2°}{K}$$

$$\boxed{K = 0.2 km}$$

Example 4: Two students are holding tethers to a helium-filled balloon that is floating directly above the line through the students' feet. Alex's tether is 25 m long at an angle of inclination of 45°. Carmen's tether is 20 m long. Determine the distance between Alex and Carmen.

Assignment: Pg. 108 # 6, 8, 22 + Worksheet