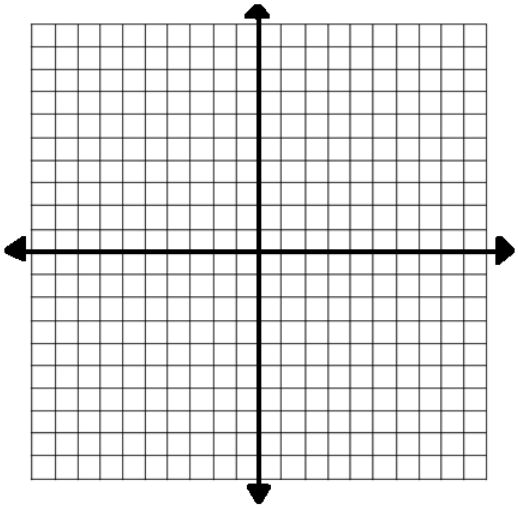
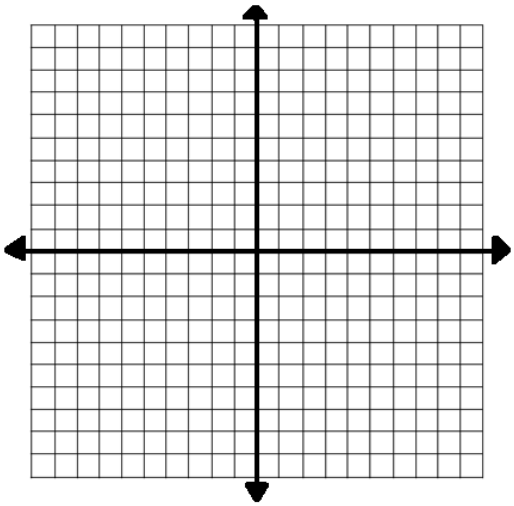
FOM 11 **Graphing Linear Functions - Review**

Sketch the graph of each line

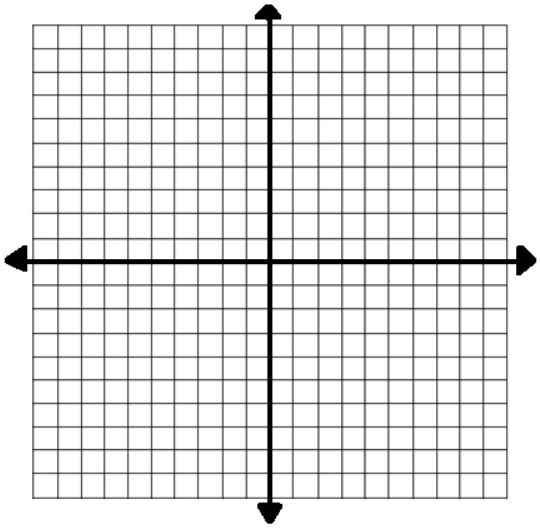
1. $y = \frac{3}{5}x - 6$



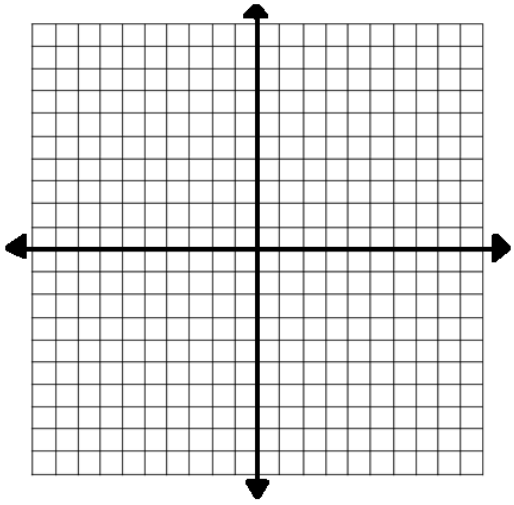
2. $y = -4x + 1$



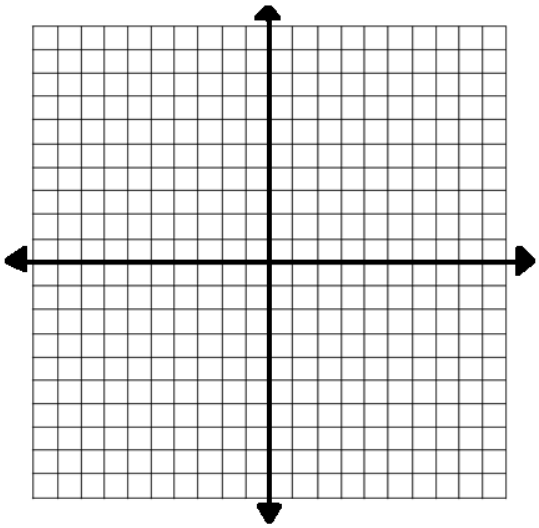
3. $3y - x = 12$



4. $2x + 7y = -\frac{35}{2}$

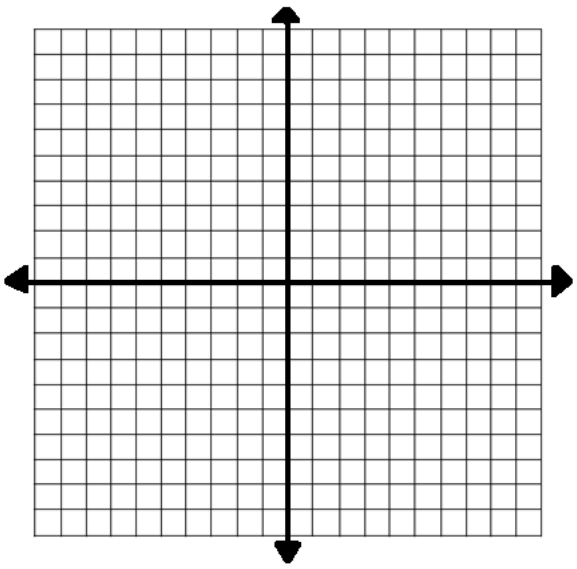


5. a) Graph the linear function that goes through (4, 3) and is perpendicular to $y = -\frac{2}{3}x - 11$



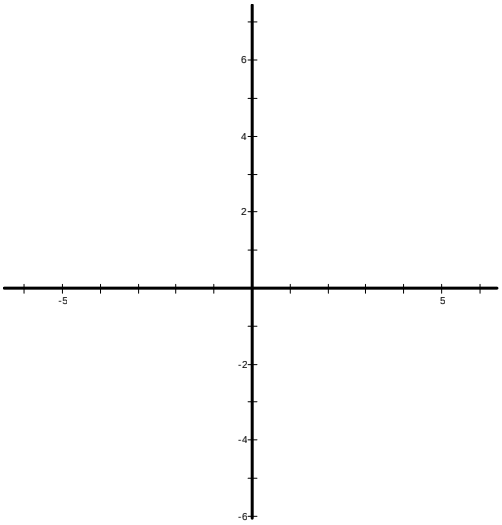
b) What is the equation of this line?

6. A company does custom paint jobs on cars and trucks. Due to the size of the workshop, and the time it takes for each job to daily output for the company is 7 vehicles in one day. Write a linear equation to model this information and sketch a graph.



FOM 11 **6.1 Graphing Linear Inequalities In Two Variables**

Look at the graph of $y = x$.



The line divides the plane into two half-planes:

- $y < x$ is the region below the line.
- $y > x$ is the region above the line.
- $y = x$ is the boundary line.

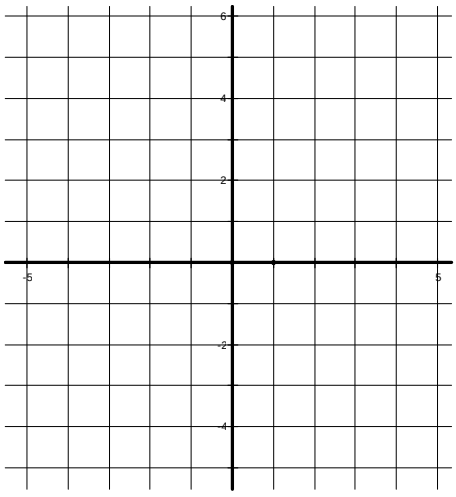
A **solid boundary line** is used to represent $\leq$ or $\geq$.

A **dotted boundary line** is used to represent < or >.

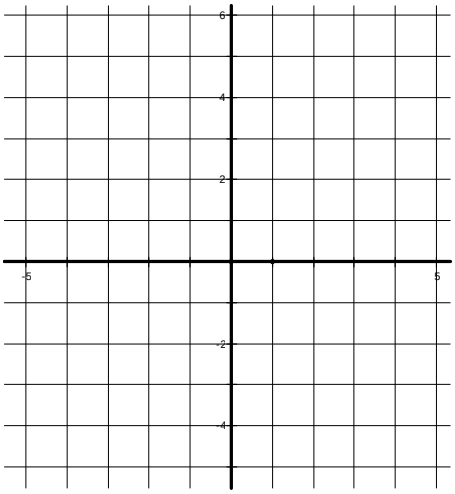
To graph an inequality:

1. Graph the boundary line.
2. Pick a point not on the line and substitute it into the inequality.
3. If the inequality is satisfied, shade the region containing the point. If not, shade the other region.

Example 1: Graph $4x - 5y < 20$.

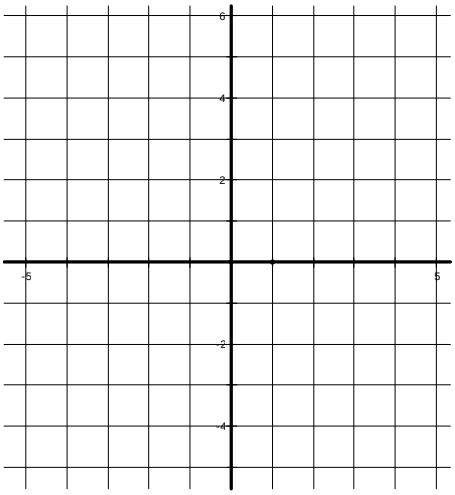


Example 2: Graph $2x - 5y \geq 10$.

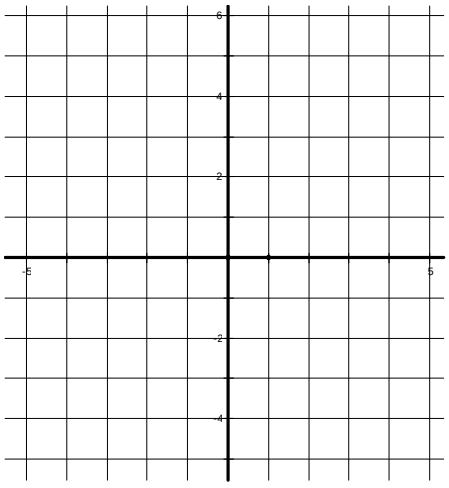


Example 3: Graph the solution set for each linear inequality on a Cartesian plane:

a. $\{(x, y) | x - 3 > 0, x \in R, y \in R\}$

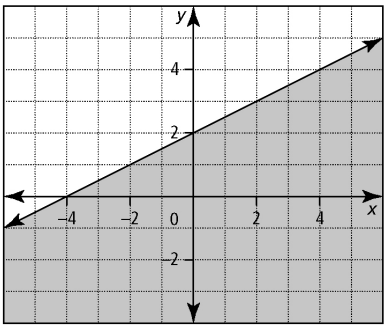


b. $\{(x, y) | -3y + 9 \geq -3 + y, x \in I, y \in I\}$

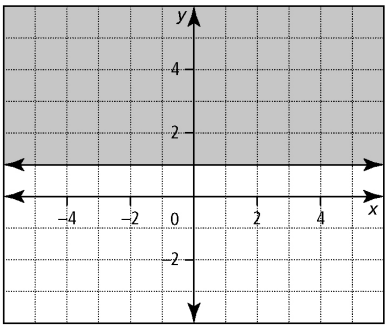


Example 4: Write an inequality to represent the graph:

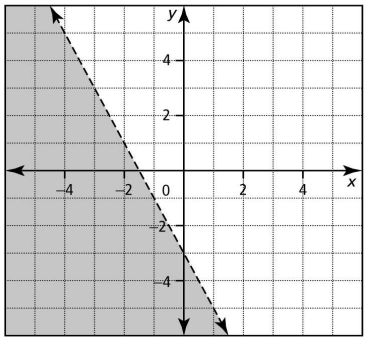
a.



b.



c.



Example 5: Ben is buying snacks for his friends. He has $10.00. The choices are apples for $0.80 and muffins for $1.25.

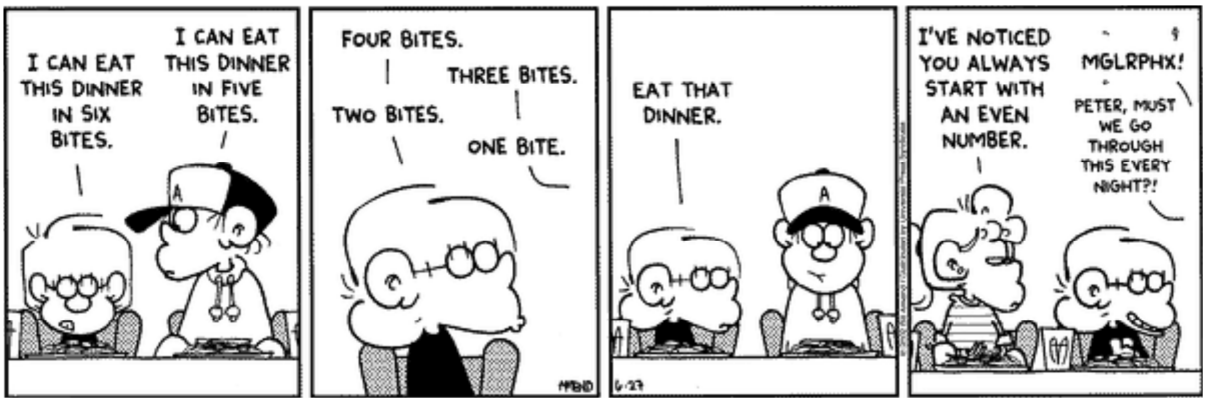
a) Write an inequality in two variables to model this situation. Define your variables.

b) State the restrictions on the variables.

c) Graph the inequality.

d) Why is (5, 4.8) not a solution?

Assignment: pg. 303 #2, 5, 6ace, 7, 8, 10-12

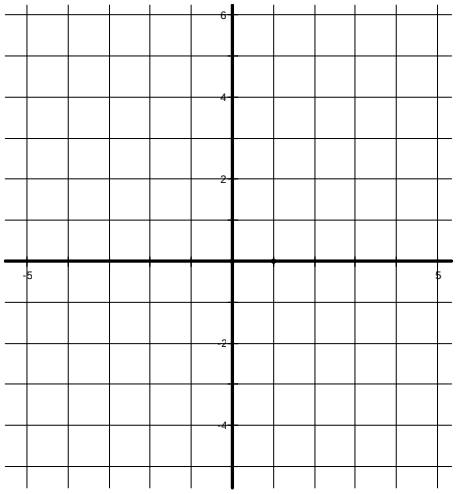


FOM 11 **6.3 Graphing to Solve Systems of Linear Inequalities**

In Grade 10 you studied systems of linear equations in two variables. The solution is the point(s) of intersection of the lines.

Eg. Solve the linear system by graphing:

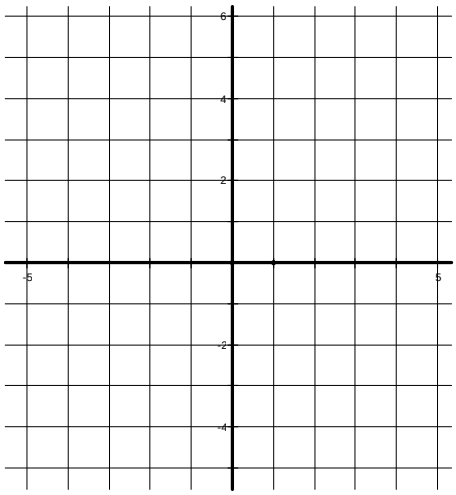
$$2x - y = -1$$
$$x + y = -2$$



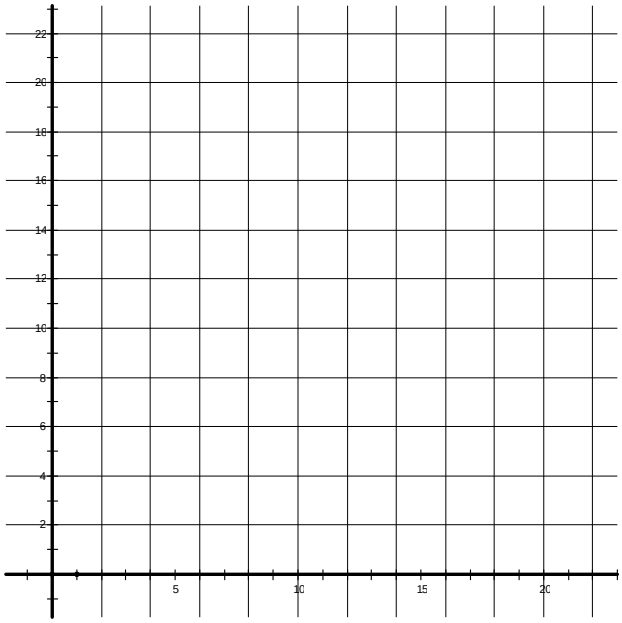
Here, we look at systems of linear **inequalities**. The solution is the region where the corresponding half-planes intersect or overlap.

Eg. Graph the region defined by these inequalities:

$$x \geq 0$$
$$y \geq 1$$
$$2x + y < 4$$



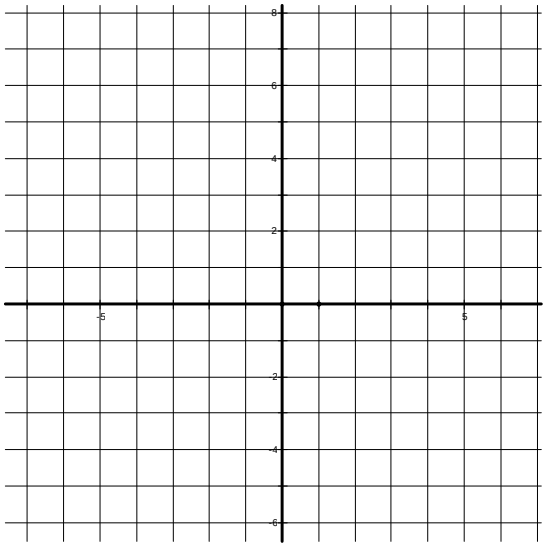
Example 1: A sporting goods manufacturer makes footballs and soccer balls. Each football takes 3 min on a cutting machine and 1 min on a stitching machine. A soccer ball takes 3 min on a cutting machine and 4 min on a stitching machine. What combinations of balls can be made in 1 hour or less?



Example 2: Graph the solution set for the following system of inequalities. Choose two possible solutions from the set.

$x + 2y < 8$

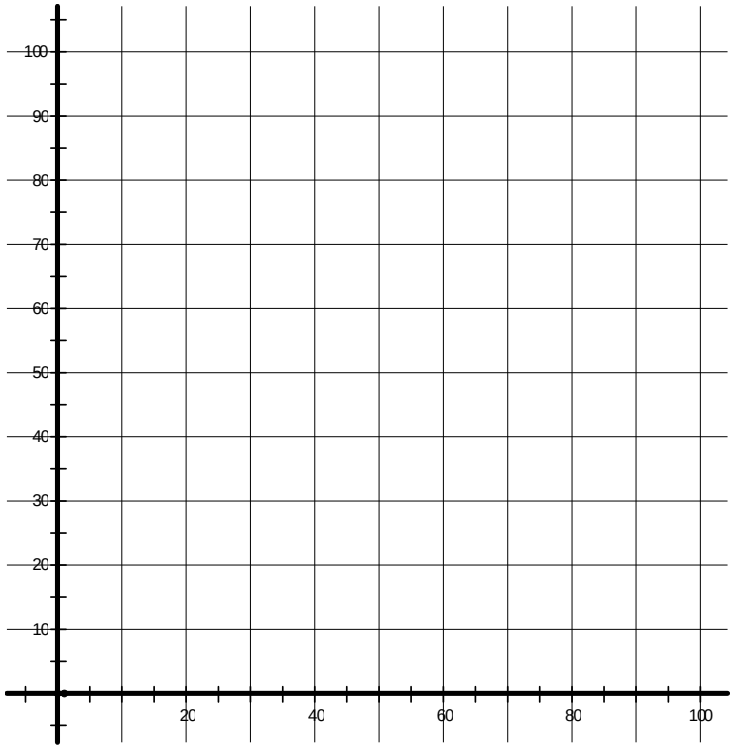
$3x - y \leq -6$



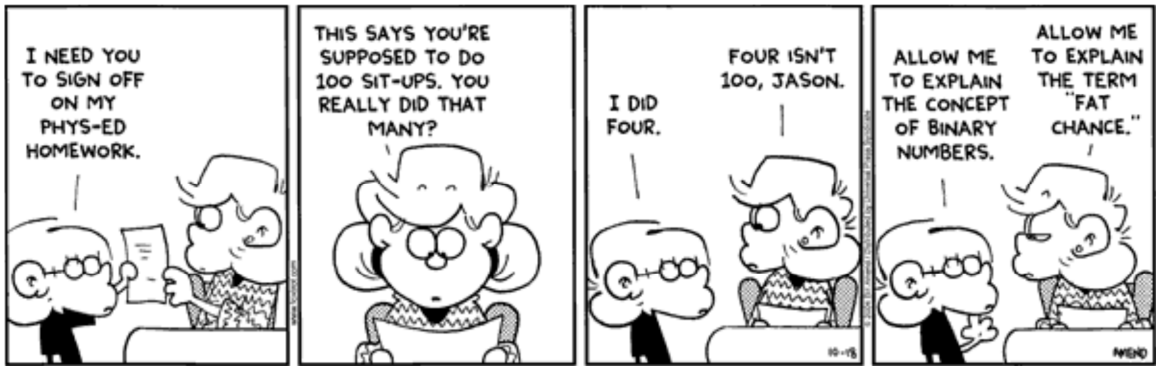
Example 3: A sloop is a sailboat with two sails: a mainsail and a jib. When a sail is fully out or up, it is said to be "out 100%". When the winds are high, sailors often reef, or pull in, the sails to be less than their full capability.

- Jim is sailing in winds of 22 knots, so he wants no more than 90% of the jib out.
- He wants to have at least twice as much jib as mainsail out.

What possible combinations of mainsail and jib can Jim have out?



Assignment: pg. 317 #4ac, 6-10, 12



FOM 11 **6.4 Optimization Problems I: Creating the Model**

Linear inequalities can be used to solve **optimization problems**, problems in which we find the greatest or least value of functions. The method used to solve such problems is called **linear programming**, and consists of two parts:

1. An **objective function** tells us the quantity we want to maximize or minimize.
2. The system of constraints consists of linear inequalities whose region is called the **feasible solution** with area called the **feasible region**.

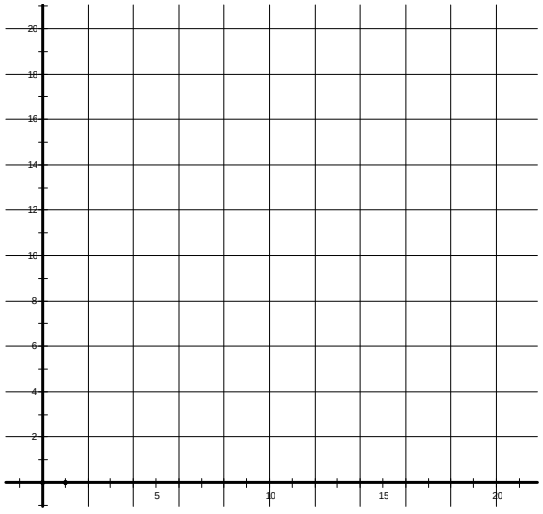
Example 1: A company makes motorcycles and bicycles. A restricted work area limits the numbers of vehicles that can be made in one day: no more than 10 motorcycles can be made, no more than 15 bicycles can be made, and no more than 20 vehicles of both kinds can be made. If the profit is $25 for a motorcycle and $50 for a bicycle, what should be the daily rate of production of both vehicles to maximize the profits?

Step 1: Define the variables that affect the quantity to be optimized and state any restrictions.

Step 2: Identify the quantity that must be optimized.

Step 3: Write an objective function.

Step 4: Write a system of linear inequalities to describe all the constraints of the problem and graph the feasible solution.



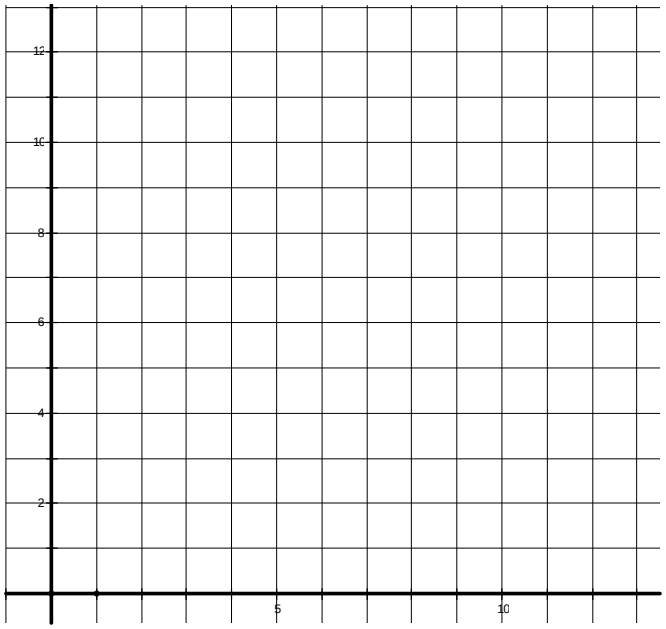
Example 2: Fred is planning an exercise program where he wants to run and swim every week. He doesn't want to spend more than 12 hours a week exercising and he wants to burn at least 1600 calories a week. Running burns 200 calories an hour and swimming burns 400 calories an hour. Running costs $1 an hour while swimming costs $2 an hour. How many hours should he spend at each sport to keep his costs at a minimum?

Step 1: Define the variables that affect the quantity to be optimized and state any restrictions.

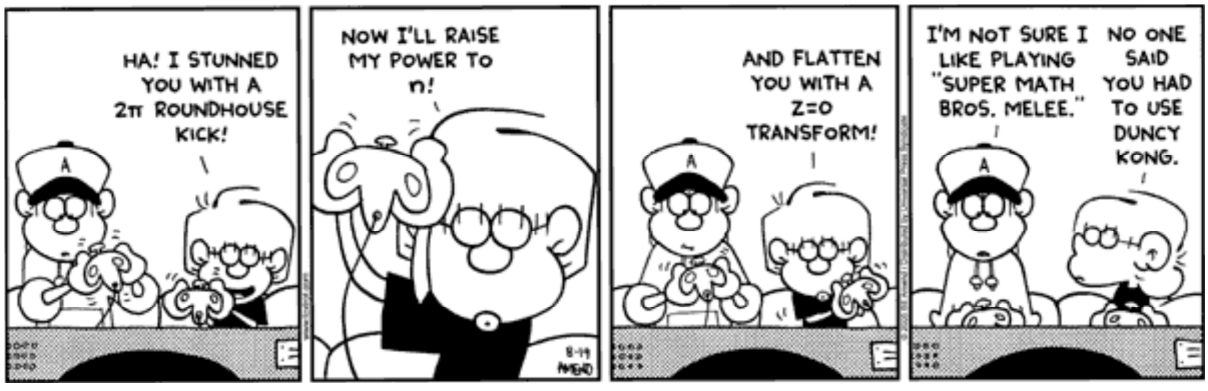
Step 2: Identify the quantity that must be optimized.

Step 3: Write an objective function.

Step 4: Write a system of linear inequalities to describe all the constraints of the problem and graph the feasible solution.



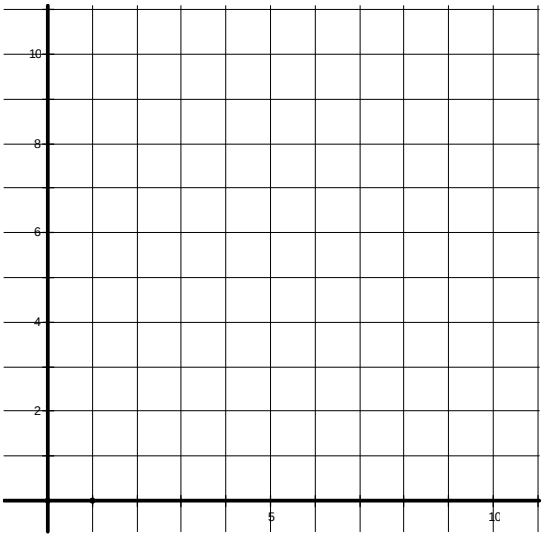
Assignment: pg. 330 #1-7



FOM 11 **6.5 Optimization Problems II: Exploring Solutions**

From last day we learned how to create a model for an optimization problem. Here we will explore the patterns in a feasible region to predict where the maximum and minimum values of an objective functions will occur.

Example 1: A company does custom paint jobs on cars and trucks. Due to the size of the workshop, the company can paint a maximum of 8 cars and 5 trucks in one day. The total output for the shop cannot exceed 10 vehicles in one day. The company earns $400 for a truck paint job and $250 for a car paint job. How many of each should they book to earn the greatest profit in one day?



Find the value of the profit throughout the feasible region and state any pattern you notice.

What happens to the value of the profit as you move from left to right?

What happens to the value of the profit as you move from the bottom to the top?

What points in the feasible region result in each **optimal solution?**

a. the maximum possible value of the profit?

b. the minimum possible value of the profit?

Summary:

- The optimal solutions to the objective function are represented by the **vertices** (or intersections of the boundaries) of the feasible region. If one or more of the intersecting boundaries is not part of the solution set, the optimal solution will be nearby.
- You can verify each optimal solution to make sure it satisfies each constraint by substituting the values of its coordinates into each linear inequality in the system.

Assignment: pg. 334 #1-3

FOM 11 **6.6 Optimization Problems III: Linear Programming**

The solution to an optimization problem is usually found at one of the vertices of the feasible region. To determine the optimal solution to an optimization problem using linear programming, follow these steps:

1. Create an algebraic model that includes:
    - A defining statement of the variables used in you model.
    - Restrictions on the variables.
    - A system of linear inequalities that describe the constraints.
    - An objective function that shows how the variables are related to the quantity being optimized.
2. Graph the system of inequalities to determine the coordinates of the vertices of the feasible region.
3. Evaluate the objective function by substituting the values of the coordinates of each vertex.
4. Compare the results and choose the desired solution.
5. Verify your solution(s) satisfies the constraints of the problem situation.

Example 1: A local craft shop produces copper bracelets and necklaces. Each bracelet requires 15 min of cutting time and 10 min of polishing time. Each necklace requires 15 min of cutting time and 20 min of polishing time. There are a maximum of 225 min of cutting time and 200 min of polishing time available each day. The shop makes a profit of $5 on each bracelet and $7 on each necklace sold. How many of each should they make per day to earn the most money?

Example 2: Fertilizer for a lawn comes in two brands as follows:

	Brand A (kg per bag)	Brand B (kg per bag)
Nitrogen	30	20
Phosphoric acid	2	4
Potash	1	4

A lawn needs at least 120 kg nitrogen, at least 16 kg of phosphoric acid, and at least 12 kg of potash. Brand A costs $22 a bag and Brand B $18 per bag. How many of bags of each brand should be used to minimize the cost? What is the minimum cost?

Assignment: pg. 341 #1, 4, 5, 11-15