7.4 Factored Form of a Quadratic Function

Remember, any quadratic equation can be written in the standard form of a quadratic $ax^2 + bx + c = 0$ where $a \neq 0$. If this factors easily, we can use the zero product theorem to extract the roots (i.e., x-intercepts).

**Zero Product Theorem:** If $ab = 0$, then $a = 0$ or $b = 0$.

Eg. $(x + 5)(x - 2) = 0$ means $x + 5 = 0 \Rightarrow x = -5$ or $x - 2 = 0 \Rightarrow x = 2$.

We can use factoring or partial factoring to help us sketch a quadratic function given in standard form.

**Example 1:** Sketch the graph of $y = 2x^2 + 12x + 10$ and state the domain and range of the function.

\[
\begin{align*}
\text{Solve} & \\
y &= 2x^2 + 12x + 10 \\
0 &= 2x^2 + 12x + 10 \\
0 &= 2(x^2 + 6x + 5) \\
0 &= 2(x^2 + 12x + 5x + 5) \\
0 &= 2[x(x+1) + 5(x+1)] \\
0 &= 2(x+1)(x+5) \\
\div 2 & \quad \div 2 \\
0 &= (x+1)(x+5) \\
x+1 = 0 & \quad x+5 = 0 \\
x = -1 & \quad x = -5
\end{align*}
\]

\[
\begin{align*}
\text{Vertex: half} & \\
x &= \frac{(-1) + (-5)}{2} = -3 \\
(-3, -8) & \\
y &= 2(-3)^2 + 12(-3) + 10 = -8
\end{align*}
\]
Example 2: Sketch the graph of $f(x) = -x^2 - 3x + 12$ and state the domain and range.

Example 3: Determine the equation of the function that defines each graph. Write each function in standard form.

a.
Example 4: A career and technology class at a high school in British Columbia operates a small T-shirt business out of the school. Over the last few years, the shop has had monthly sales of 300 T-shirts at a price of $15 per T-shirt. The students have learned that for every $2 increase in price, they will sell 20 fewer T-shirts each month. What should they charge for their T-shirts to maximize their monthly revenue?

Assignment: pg. 391 #2, 3, 8, 9, 10ace, 11, 16