FOM 11

7.2 Properties of Graphs Of Quadratic Functions

Remember that the vertex of a quadratic function is the maximum or minimum point of the parabola. To find the vertex from an equation we can use the symmetry of the parabola to help us.

(axis of symmetry will be half-way between 2 identical y-values)

Example 1: Find the vertex of the following quadratic functions, and state if the vertex is a maximum or a minimum.

a. \( y = x^2 - 2x + 4 \) 

\[
\begin{array}{c|c}
 x & y \\
-2 & 12 \\
-1 & 7 \\
0 & 4 \\
1 & 3 \\
2 & 4 \\
\end{array}
\]

vertex (1, 3)

\( (1 + 2) \div 2 = 1.5 \) 
\( x = 1.5 \)
\( y = -(1.5)^2 + 3(1.5) - 4 \)
\( = -1.75 \)

vertex (1.5, -1.75)

b. \( y = -x^2 + 3x - 4 \)

\[
\begin{array}{c|c}
 x & y \\
-2 & -4 \\
-1 & -4 \\
0 & 0 \\
1 & 2 \\
2 & 20 \\
\end{array}
\]

vertex (1, 2)

\( (-2 + -1) \div 2 = -1.5 \) 
\( x = -1.5 \)
\( y = 2(-1.5)^2 + b(-1.5) \)
\( = -4.5 \)

vertex (-1.5, -4.5)

c. \( y = 2x^2 + 6x \)

\[
\begin{array}{c|c}
 x & y \\
-2 & -14 \\
-1 & -8 \\
0 & 0 \\
1 & -2 \\
2 & 4 \\
\end{array}
\]

e. \( y = 3x^2 - 6x + 2 \)

vertex (-1, -3.5)

Example 2: A water arch at a splash pad is defined by the quadratic function:

\( f(x) = -0.15x^2 + 3x \).

Graph the function, and state its domain and range.
Example 3: Sketch the graph of the given function, then find the desired properties:

a. \( y = -x^2 - 2x + 3 \)
   i. Vertex: \((-1, 4)\)
   ii. Y-intercept: \((0, 3)\)
   iii. X-intercept: \((-3, 0), (1, 0)\)
   iv. Axis of symmetry: \(x = -1\)
   v. Domain: \(x \in \mathbb{R}\)
   vi. Range: \(y \leq 4\)

\[ \text{Maximum } y = 4 \]

b. \( y = x^2 + x - 2 \)
   i. Vertex: \((-0.5, -2.25)\)
   ii. Y-intercept: \((0, -2)\)
   iii. X-intercept: \((-2, 0), (1, 0)\)
   iv. Axis of symmetry: \(x = -0.5\)
   v. Domain: \(x \in \mathbb{R}\)
   vi. Range: \(y \geq -2.25\)

\((-1+0)/2 = -0.5\)
\[ y = (-0.5)^2 + (0.5) - 2 = -2.25 \]
Example 4: Some boaters use red aerial miniflares in an emergency. The path of one brand of flare, when fired at an angle of 70° to the horizontal, is modeled by the function:

\[ h(t) = -9(t - 3)^2 + 83 \]

where \( h(t) \) is the height in metres and \( t \) is the time in seconds since the flare was fired.

a. Sketch a graph of the function, including your window.

[Graph showing the function with a maximum point at (3.83, 83)]

b. What is the maximum height of the flare?

83 m

c. How many seconds until the flare hits the water?

6.04 seconds

d. If the flare burns red for 2 seconds, how high is it when it burns out?

\[ h(2) = -9(2-3)^2 + 83 \]
\[ = -9(-1)^2 + 83 \]
\[ = -9 + 83 \]
\[ = 74 \text{ m} \]

Assignment: pg 368 #1, 4-7, 9-11, 12-14

p. 369 #4, 11, 13