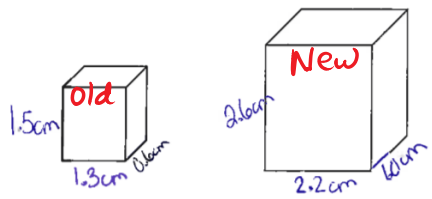
8.5 Similar Objects: Scale Models and Scale Diagrams

We have learned how to draw scale diagrams in 2-D using a scale factor. We can use a similar strategy when dealing with 3-D objects as well.

Example 1: Are the two figures similar? Justify your answer.



$\frac{2.6\text{cm}}{1.5\text{cm}} = 1.73 \approx 1.7$ $\quad \frac{2.2\text{cm}}{1.3\text{cm}} = 1.69 \approx 1.7$ $\quad \frac{1.0\text{cm}}{0.6\text{cm}} = 1.67 \approx 1.7$

∴ They are similar b/c each dimension has the same scale factor.

Example 2: Sarah bought a toy tractor to give to her younger brother for his birthday. The dimensions of the toy are given in the diagram. The scale ratio on the package is 1:16. What are the dimensions of the real tractor?

$\frac{12.7\text{cm}}{x} = \frac{1}{16}$

$x = 12.7 \times 16$

$x = 203.2\text{ cm}$

$\frac{9.5\text{cm}}{x} = \frac{1}{16}$

$x = 9.5 \times 16$

$x = 152\text{cm}$

$\frac{19.1\text{cm}}{x} = \frac{1}{16}$

$x = 16 \times 19.1$

$x = 305.6\text{cm}$

8.6 Scale Factors and 3-D Objects

We have looked at the relationship between the scale factor and the area of 2-D objects. Here we will look at the relationship between the scale factor and the surface area and volume of 3-D objects.

Complete the following table:

$SA = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Sphere	Radius (cm)	Surface Area (cm^2)	Volume (cm^3)
A	1	12.57 cm^2	4.1888 cm^3
B	3	113.10 cm^2	113.0973 cm^3
C	5	314.16 cm^2	523.5988 cm^3

Use the data from the table above to complete the following:

Image : Object	Scale Factor	Ratio of Surface Areas	Ratio of Volumes
B to A	$\frac{3}{1} = 3$	$\frac{113.10}{12.57} = 9$	$\frac{113.0973}{4.1888} = 27$
C to A	$\frac{5}{1} = 5$	$\frac{314.16}{12.57} = 25$	$\frac{523.5988}{4.1888} = 125$
C to B	$\frac{5}{3} = 1.\overline{6}$	$\frac{314.16}{113.10} = 2.\overline{7}$	$\frac{523.5988}{113.0973} = 4.63$

What relationship is there between the scale factor and the ratio of the surface areas of the two spheres?

$$\text{Object S.A.} \times SF^2 = \text{Image S.A.}$$

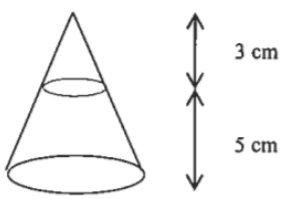
$$(\text{Scale factor})^2 = \frac{\text{Image S.A.}}{\text{object S.A.}}$$

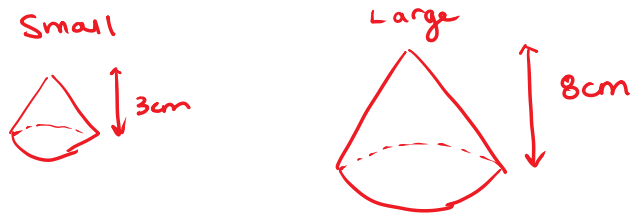
What relationship is there between the scale factor and the ratio of the volumes of the two spheres?

$$\text{Object Vol.} \times SF^3 = \text{Image Vol.}$$

$$(\text{Scale Factor})^3 = \frac{\text{Image Vol.}}{\text{object Vol.}}$$

Example 1: A plane parallel to the base of a cone divides the cone into two pieces. Find the ratio of:





a) The surface area of the ~~small~~ large cone to the surface area of the ~~large~~ small cone.

$$\frac{\text{Image SA}}{\text{Object SA}} = (\text{Scale factor})^2 = \left(\frac{8}{3}\right)^2 = \frac{8^2}{3^2} = \frac{64}{9}$$

b) The volume of the large cone to the volume of the small cone.

$$\frac{\text{Image Vol.}}{\text{Object Vol.}} = (\text{Scale factor})^3 = \left(\frac{8}{3}\right)^3 = \frac{512}{27}$$

Homework: p. 497 # 1 (b, c), 2, 4, 6, 8, 11*, 14* and p. 508 # 1-3, 6, 10, 11*, 14*

*Optional