8.5 Similar Objects: Scale Models and Scale Diagrams

We have learned how to draw scale diagrams in 2-D using a scale factor. We can use a similar strategy when dealing with 3-D objects as well.

Example 1: Are the two figures similar? Justify your answer.

\[
\begin{align*}
\frac{2.6\text{ cm}}{1.5\text{ cm}} &= 1.73 \\
\frac{2.2\text{ cm}}{1.3\text{ cm}} &= 1.69 \\
\frac{1.0\text{ cm}}{0.6\text{ cm}} &= 1.67
\end{align*}
\]

\[\therefore \text{ They are similar because each dimension has the same scale factor.}\]

Example 2: Sarah bought a toy tractor to give to her younger brother for his birthday. The dimensions of the toy are given in the diagram. The scale ratio on the package is 1:16. What are the dimensions of the real tractor?

\[
\begin{align*}
\frac{12.7\text{ m}}{x} &= \frac{1}{16} \\
\frac{9.5\text{ cm}}{x} &= \frac{1}{16} \\
\frac{19.1\text{ cm}}{x} &= \frac{1}{16}
\end{align*}
\]

\[
\begin{align*}
x &= 12.7 \times 16 \\
x &= 9.5 \times 16 \\
x &= 19.1 \times 16
\end{align*}
\]

\[
\begin{align*}
x &= 203.2 \text{ cm}
\end{align*}
\]

8.6 Scale Factors and 3-D Objects

We have looked at the relationship between the scale factor and the area of 2-D objects. Here we will look at the relationship between the scale factor and the surface area and volume of 3-D objects.

Complete the following table:

<table>
<thead>
<tr>
<th>Sphere</th>
<th>Radius (cm)</th>
<th>Surface Area (cm(^2))</th>
<th>Volume (cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>12.57 cm(^2)</td>
<td>4.1888 cm(^3)</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>113.10 cm(^2)</td>
<td>113.0973 cm(^3)</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>314.16 cm(^2)</td>
<td>523.5988 cm(^3)</td>
</tr>
</tbody>
</table>

Use the data from the table above to complete the following:

<table>
<thead>
<tr>
<th>Image : Object</th>
<th>Scale Factor</th>
<th>Ratio of Surface Areas</th>
<th>Ratio of Volumes</th>
</tr>
</thead>
<tbody>
<tr>
<td>B to A</td>
<td>(\frac{3}{1} = 3)</td>
<td>(\frac{113.10}{12.57} = 9)</td>
<td>(\frac{113.0973}{4.1888} = 27)</td>
</tr>
<tr>
<td>C to A</td>
<td>(\frac{5}{1} = 5)</td>
<td>(\frac{314.16}{12.57} = 25)</td>
<td>(\frac{523.5988}{4.1888} = 125)</td>
</tr>
<tr>
<td>C to B</td>
<td>(\frac{5}{3} = 1.6)</td>
<td>(\frac{314.16}{113.10} = 2.7)</td>
<td>(\frac{523.5988}{113.0973} = 4.63)</td>
</tr>
</tbody>
</table>
What relationship is there between the scale factor and the ratio of the surface areas of the two spheres?

\[
\text{Object S.A.} \times SF^2 = \text{Image S.A.}
\]

\[
(Scale \text{ factor})^2 = \frac{\text{Image S.A.}}{\text{Object S.A.}}
\]

What relationship is there between the scale factor and the ratio of the volumes of the two spheres?

\[
\text{Object Vol.} \times SF^3 = \text{Image Vol.}
\]

\[
(Scale \text{ Factor})^3 = \frac{\text{Image Vol.}}{\text{Object Vol.}}
\]

Example 1: A plane parallel to the base of a cone divides the cone into two pieces. Find the ratio of:

\[\text{Small cone} \quad \text{Large cone}\]

\[
\text{a) The surface area of the small cone to the surface area of the large cone.}
\]

\[
\frac{\text{Image SA}}{\text{Object SA}} = (Scale \text{ factor})^2 = \left(\frac{8}{3}\right)^2 = \frac{8^2}{3^2} = \frac{64}{9}
\]

\[
\text{b) The volume of the large cone to the volume of the small cone.}
\]

\[
\frac{\text{Image Vol.}}{\text{Object Vol.}} = (Scale \text{ factor})^3 = \left(\frac{8}{3}\right)^3 = \frac{512}{27}
\]

Homework: p. 497 # 1 (b, c), 2, 4, 6, 8, 11*, 14* and p. 508 # 1-3, 6, 10, 11*, 14*

*Optional