8.3 Scale Diagrams

A **scale drawing** is a reproduction of a diagram. It is either larger or smaller than the original, but it has the **same shape** (ie. It is **similar**). Each dimension of the figure is multiplied by the same **scale factor**.

In order to visualize the actual figure, you need to know the scale of the scale drawing. You will need to use ratios and convert measurement units.

The scale of a diagram can be specified in four ways:

1. as a statement (1cm = 50km)
2. as a rate $\left(\frac{1\text{cm}}{50\text{km}}\right)$
3. as a ratio (1 : 5000 000)
4. as a linear scale

If the image length is larger than the object length, then the scale factor is greater than 1 and we have an **enlargement**. If the image length is less than the object length, then the scale factor is less than 1 and we have a **reduction**.

$$\text{Scale factor} = \frac{\text{Image}}{\text{object}} \quad \left(\frac{\text{New}}{\text{old}}\right)$$

Example 1: Complete the table.

Object Length	Image Length	Scale Factor	Enlargement or Reduction
7cm	14cm	$\frac{14}{7} = 2$	Enlargement
15 in	5 in	$\frac{5}{15} = \frac{1}{3}$	Reduction
4m	6m	1.5	Enlargement
8ft	3.2ft	0.4	Reduction

$\frac{6}{x} = \frac{1.5}{1}$

$1.5x = 6$

$x = 4$

$\frac{x}{8} = \frac{0.4}{1}$

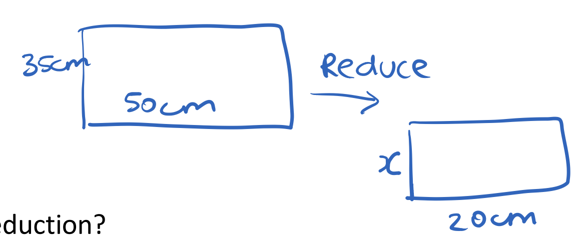
$1x = 0.4 \times 8$

$x = 3.2$

Example 2: A portrait measures 50cm by 35cm. The larger side of a reduction of its measures 20cm.

a) What is the scale factor of the reduction?

$$\frac{\text{New}}{\text{old}} = \frac{20\text{cm}}{50\text{cm}} = \frac{2}{5} = 0.4$$



b) What is the length of the shorter side of the reduction?

$$\frac{\text{New}}{\text{old}} \quad \frac{x}{35} = \frac{0.4}{1}$$

$1x = 0.4 \times 35$

$x = 14$

14cm.

8.4 Scale Factors and Areas of 2-D Shapes

8.3 – we studied the concept of **similar objects** – objects which have the same shape by different sizes. Another way of describing the enlargement or reduction of an object proportionally is by **scale factors**. Here we explore the relationship between scale factors and the **perimeter** and **area** of 2-D objects.

Complete the table below for similar rectangles:

Rectangle	Length (cm)	Width (cm)	Perimeter (cm)	Area (cm^2)
A	3cm	2cm	10 cm	6 cm^2
B	6cm	4cm	20cm	24cm^2
C	9cm	6cm	30 cm	54 cm^2

Use the results from the table above to complete the following table:

Rectangle	Scale Factor	Ratio of Perimeters	Ratio of Areas
B to A	$\frac{6}{3} = 2$	$\frac{20}{10} = 2$	$\frac{24}{6} = 4$
C to A	$\frac{9}{3} = 3$	$\frac{30}{10} = 3$	$\frac{54}{6} = 9$
C to B	$\frac{9}{6} = \frac{3}{2} = 1.5$	$\frac{30}{20} = 1.5 = \frac{3}{2}$	$\frac{54}{24} = \frac{27}{12} = \frac{9}{4} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$

What is the relationship between the scale factor and the ratios of the perimeters?

scale factor is equal to the ratio of the perimeters.

What is the relationship between the scale factor and the ratio of the areas?

$$\text{Ratio of area} = (\text{Scale factor})^2$$

Example 1: Jasmine is making a kite from a 1:20 scale diagram. The area of the scale diagram is 30 cm^2. How much fabric will she need for her kite?

$$\left(\frac{1}{20}\right)^2 = \frac{30\,cm^2}{x}$$

$$\frac{1}{400} = \frac{30}{x}$$

$$1x = 30 \times 400$$

$$x = 12000$$

$\boxed{\text{Jasmine needs } 12{,}000\,cm^2 \text{ of fabric}}$

Homework: p. 479 # 1 – 4, 6 - 8, 12*, 14* and p. 487 # 1 - 3, 4 a, 5 a, 8, 9, 10*, 13*

*Optional