

1.4.2

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When we make a conclusion based on statements that we accept as true, we are using **deductive reasoning**. The rules we follow when performing algebraic manipulations are things that we accept (and know) as true. So we are using deductive reasoning to prove a statement is always true.

Statements that we know are true:

Any integer multiplied by 2 is an even number. $2x$

- This means that $2x$ or $2(\text{any combination of variables and coefficients})$ will **always** be even.

If you add 1 to any even integer you will get an odd number. $2x+1$

- This means that $2x + 1$ or $2(\text{any combination of variables and coefficients}) + 1$ will **always** be odd.

Consecutive Numbers follow each other in numerical order

- This means that $x, x + 1, x + 2, x + 3$ are 4 numbers that come one after the other numerically.
- $2x, 2x + 2, 2x + 4, 2x + 6$ are 4 consecutive even numbers
- $2x + 1, 2x + 3, 2x + 5, 2x + 7$ are 4 consecutive odd numbers

Example 1: Use deductive reasoning to prove that the sum of an odd number and an even number is always odd.

$$\begin{aligned}
 & \text{add} \quad \frac{2x+1}{\quad} \quad \frac{2y}{\quad} \\
 & (2x+1) + 2y \\
 & = 2x+1+2y \\
 & = 2x+2y+1 \\
 & = 2(x+y) + 1 \quad \rightarrow \text{This is even.} \\
 & = 2(\text{Some \#}) + 1 \quad \rightarrow \text{by adding 1, this is always odd.}
 \end{aligned}$$

Finishing a Proof:

- If proving an answer is even it should look like this $\rightarrow 2(\text{any combination of variable terms})$
- If proving an answer is odd it should look like this $\rightarrow 2(\text{any combination of variable terms}) + 1$
- If proving an answer is divisible by 3 it should look like this $\rightarrow 3(\text{any combination of variable terms})$
- If proving an answer is divisible by 4 it should look like this $\rightarrow 4(\text{any combination of variable terms})$
- If proving an answer is divisible by 5 it should look like this $\rightarrow 5(\text{any combination of variable terms})$
- etc.....

Example 2: Prove that the square of an even integer is always even

$$\begin{aligned}
 & (2z)^2 \\
 &= 2^2 z^2 \\
 &= 4z^2 = 2(2z^2)
 \end{aligned}$$

multiply by 2
It will be even always.

Example 3: Prove that the result of the number trick below is always the number you start with.

<ul style="list-style-type: none"> - Choose a number - Add 2 - Multiply by 3 - Subtract 6 - Subtract your original number - Divide by 2 	$ \begin{aligned} & f \\ & f+2 \\ & 3(f+2) = 3f+6 \\ & 3f+6-6 = 3f \\ & 3f-f = 2f \\ & 2f \div 2 = f \end{aligned} $	$ \begin{aligned} & 7 \\ & 9 \\ & 27 \\ & 21 \\ & 14 \\ & 7 \end{aligned} $
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Example 4: The sum of a two digit number and its reversal is a multiple of 11.

<p>pick $23 + 32$ $20+3+30+2$ $= 55$</p>	$ \begin{aligned} & xy \quad \uparrow \quad yx \\ & = 10x+y \quad = 10y+x \\ \text{Sum} & \Rightarrow 10x+y+10y+x \\ & = 10x+x+10y+y \\ & = 11x+11y \\ & = 11(x+y) \end{aligned} $ <p>\therefore Being multiplied by 11, so $11(x+y)$ is a multiple of 11.</p>
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Assignment: Deductive Reasoning Worksheet