Ans. Key

Deductive Reasoning Worksheet

Name: ________________

Statements that we know are true:

Any integer multiplied by 2 is an even number.

- This means that $2x$ or 2(any combination of variables and coefficients) will **always** be even.

If you add 1 to any even integer you will get an odd number.

- This means that $2x + 1$ or 2(any combination of variables and coefficients) + 1 will **always** be odd.

Consecutive Numbers follow each other in numerical order

- This means that $x, x+1, x+2, x+3$ are 4 numbers that come one after the other numerically.
- $2x$, $2x+2$, $2x+4$, $2x+6$ are 4 consecutive even numbers
- $2x+1$, $2x+3$, $2x+5$, $2x+7$ are 4 consecutive odd numbers

Finishing a Proof:

- If proving an answer is even it should look like this → 2(any combination of variable terms)
- If proving an answer is odd it should look like this → 2(any combination of variable terms) + 1
- If proving an answer is divisible by 3 it should look like this → 3(any combination of variable terms)
- If proving an answer is divisible by 4 it should look like this → 4(any combination of variable terms)
- If proving an answer is divisible by 5 it should look like this → 5(any combination of variable terms)
- etc……

Important Tips to remember:

- Proving Mathematical Concepts using **Deductive Reasoning** uses **ALGEBRA**
- Remember to square something means to multiply by itself! If there is more than 1 term being squared you **MUST** use the box method (or arrows) to multiply
- Sum = Add
- Difference = Subtract
- Product = Multiply
- Quotient = Divide
- **If the question involves more than one "number" you must use a different letter (variable) for each term!** (use same variable for any consecutive numbers)

For example, to prove something about the sum of 2 odd numbers and an even number it would look like this:

$$(2x + 1) + (2y + 1) + 2z$$

Odd + Odd + Even

Prove the following deductively:

1. Conjecture: The sum of five consecutive integers is always divisible by five.

(Prove the conjecture by manipulating the expression to look like this 5(........)

$$x + (x + 1) + (x + 2) + (x + 3) + (x + 4)$$

$x + x + 1 + x + 2$

$3x + 3$

$3(x+1)$

$= x + x + 1 + x + 2 + x + 3 + x + 4$

$= 5x + 10$

$= 5(x+2)$

2. Prove that the sum of two even numbers and an odd number is always odd.

(Prove the conjecture by manipulating the expression to look like this 2(........) + 1

$$2x + 2y + (2z + 1)$$

$= 2x + 2y + 2z + 1$

$= 2(x + y + z) + 1$

3. Prove that the sum of any two odd integers is even.

Let $2x+1$ be odd int.

" $2y+1$ be odd int.

$(2x+1) + (2y+1)$

$= 2x + 2y + 2$

$= 2(x + y + 1)$

4. Prove that the negative of any even integer is even.

$-(2x)$

$= -2x$

$= 2(-x)$

5. Prove that the difference between an even integer and an odd integer is odd.

$2y - 2x+1$

$= 2(y-x)+1$

6. Prove that the product of an odd integer and an even integer is always even.

$(2x+1)(2y)$

$= 4xy+2y$

$= 2(2xy+y)$

7. Prove that the result of the number trick below is always the number you start with.

- Choose a number	n	5
- Double it	$2n$	10
- Add 6	$2n+6$	16
- Double it again	$4n+12$	32
- Subtract 4	$4n+8$	28
- Divide by 4	$n+2$	7
- Subtract 2	n	5

8. Prove that whenever you square an odd integer, the result is odd.

$(2x+1)^2$

$= (2x+1)(2x+1)$

$= 4x^2+2x+2x+1$

$= 4x^2+4x+1$

$= 2(2x^2+2x)+1$

9. Prove that the sum of three consecutive integers is always a multiple of 3

$x+x+1+x+2$

$=3x+3$

$=3(x+1)$

10. Prove that the difference between the square of any odd integer and the integer itself is always an even integer.

$(2x+1)^2-(2x+1)$

$=(2x+1)(2x+1)-2x-1$

$=4x^2+2x+\cancel{2x}+\cancel{1}-\cancel{2x}-\cancel{1}$

$=4x^2+2x$

$=2(2x^2+x)$

11. Write any two digit number. Reverse the order of the digits and subtract it from the first number. (example: 81 – 18)

a) Investigate this for different starting numbers and make a conjecture

$81-18=63$

$31-13=18$

$32-23=9$

The difference of 2 digit # and its reverse is always ~~add up to be 9~~ multiple of 9.

b) Prove that the difference is always a multiple of 9.

Hint: A two digit number can be written as the sum of the digit in the tens place value, and the digit in the unit place value → a number xy is $(10x + y)$ the reverse number yx would be $(10y + x)$

$(10x+y)-(10y+x)$

$=10x+y-10y-x$

$=9x-9y$

$=9(x-y)$